

# Introduction to radio astronomy

## Lectures 5 and 6

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ALMA Regional Centre || Czech



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of the Czech Academy  
of Sciences

- ▶ Methods of radio astronomy – a quick overview
  - Radiometry, polarimetry,
  - SD imaging
  - „Active“ radioastronomy – radar sounding
  - Radio spectroscopy
  - Interferometry overview – sparse phased arrays, aperture synthesis
- ▶ Intro to (solar) radio spectroscopy
  - Meaning of radio spectra
  - Examples at metric wavelengths
  - Decimeter emission – flare related bursts
  - Calibration of spectroscopic data
  - Non-solar applications
- ▶ Intro to interferometry/aperture synthesis
  - Basic principles of Fourier imaging
  - From complex visibilities to „real“ image

All lectures to be found at: <http://wave.asu.cas.cz/barta/lectures/radioastronomy>

Brief overview/comparison of all methods

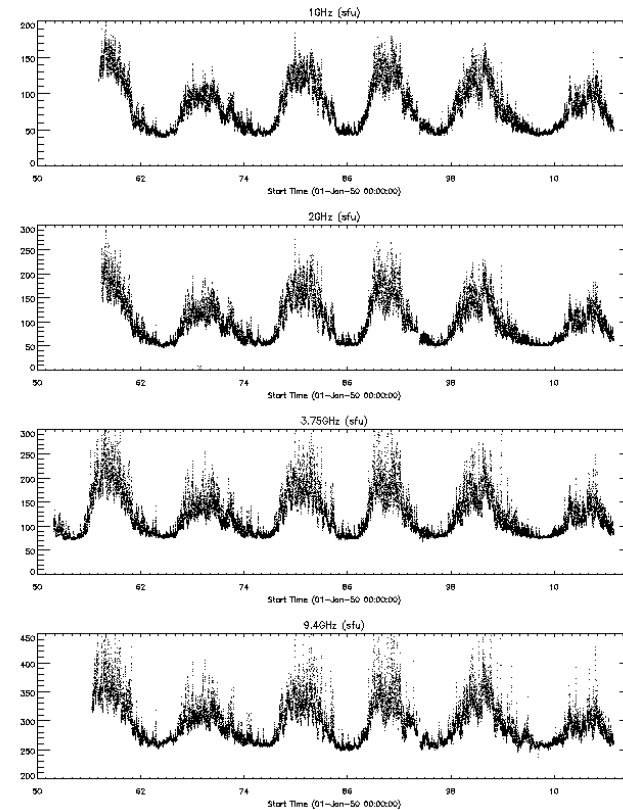
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# Methods of radio astronomy

## Radiometry

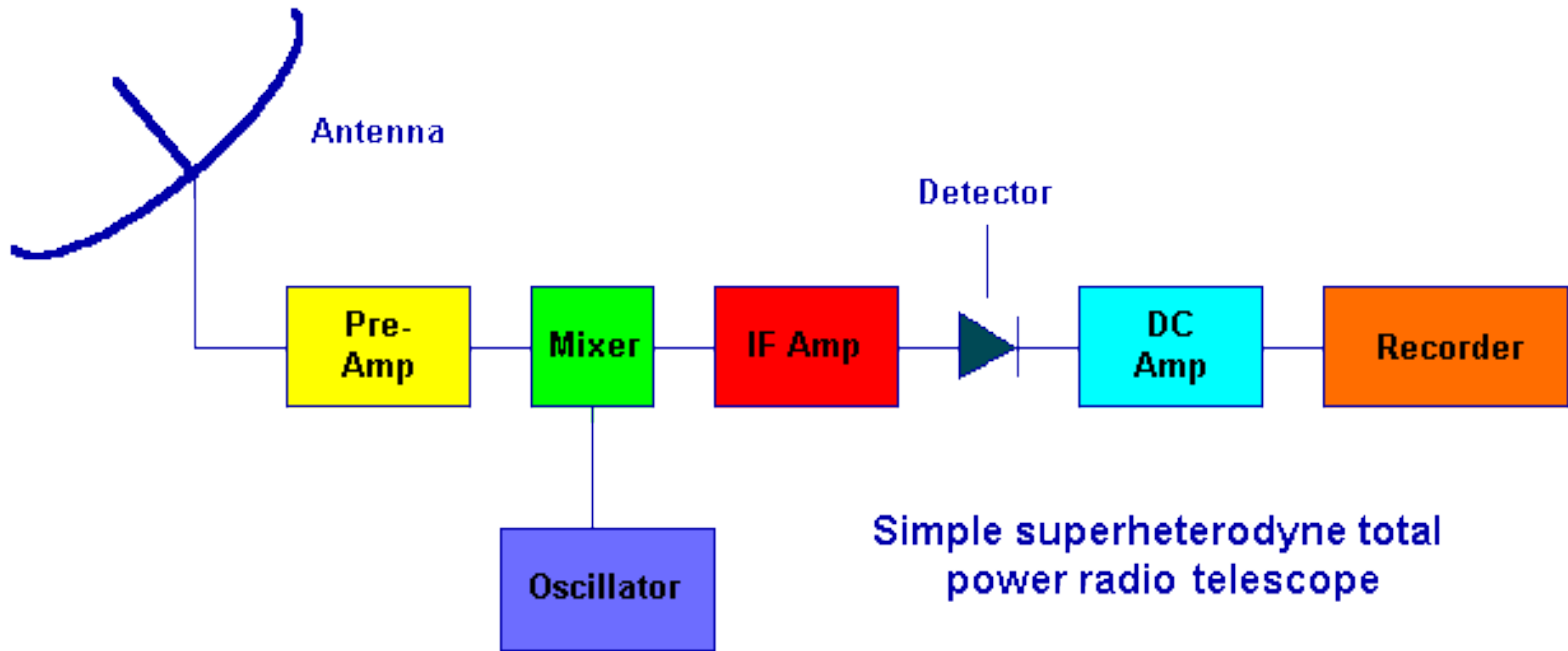


Result: A light curve  
Pros: Good time resolution  
Cons: No spatial, spectral nor pol. information



# Methods of radio astronomy

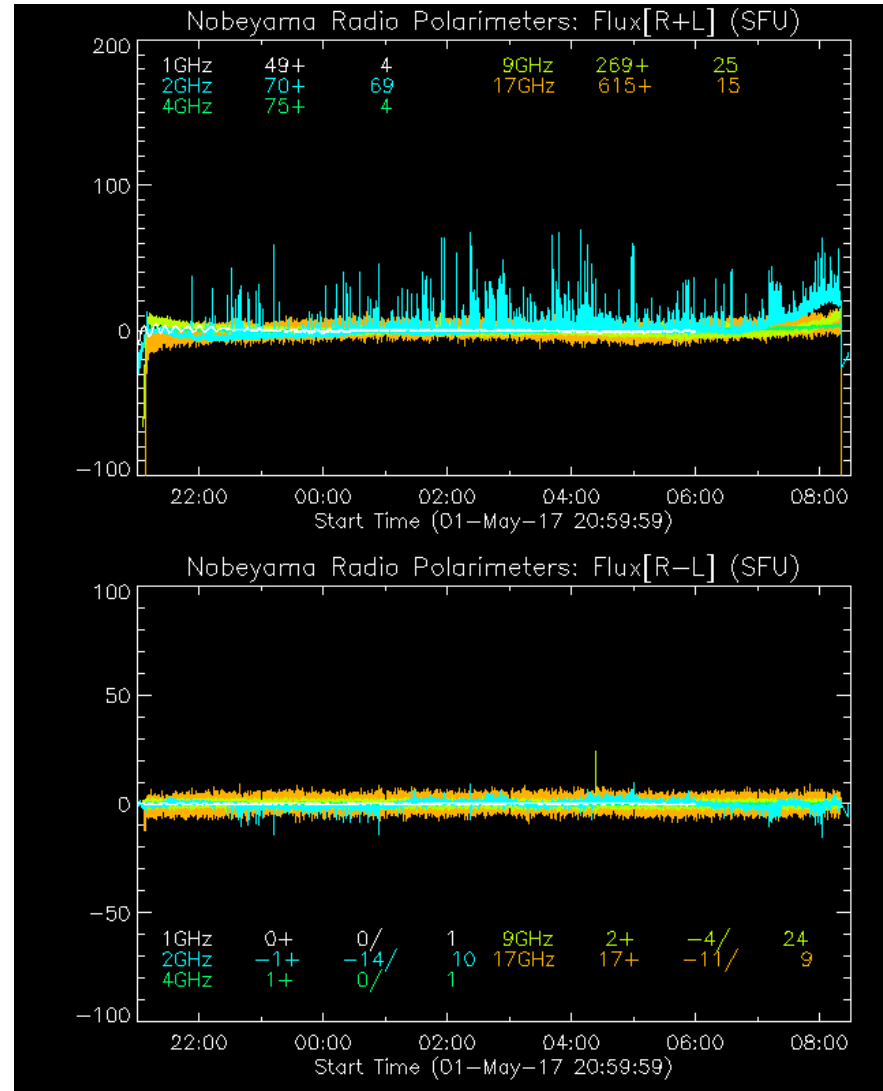
## Radiometry



# Methods of radio astronomy

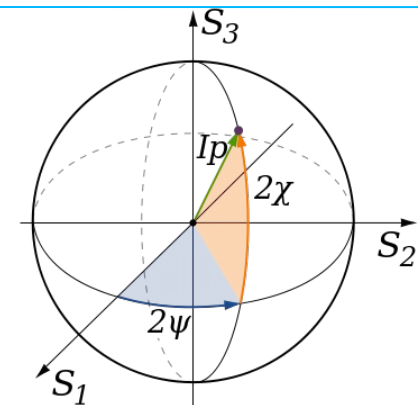
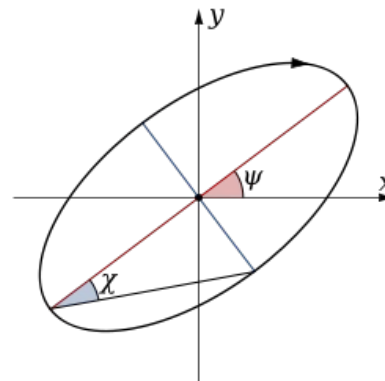
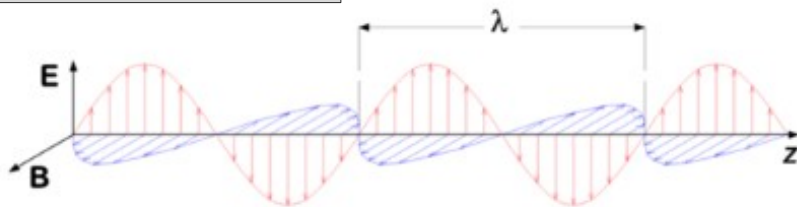
## Radio polarimetry

Result: Two light curves  
Pros: Good time resolution, polarisation  
Cons: No spatial, spectral information



# Methods of radio astronomy

## Radio polarimetry



## Polarisation: Stokes vector

$$\vec{S} = \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}$$

$$S_0 = I$$

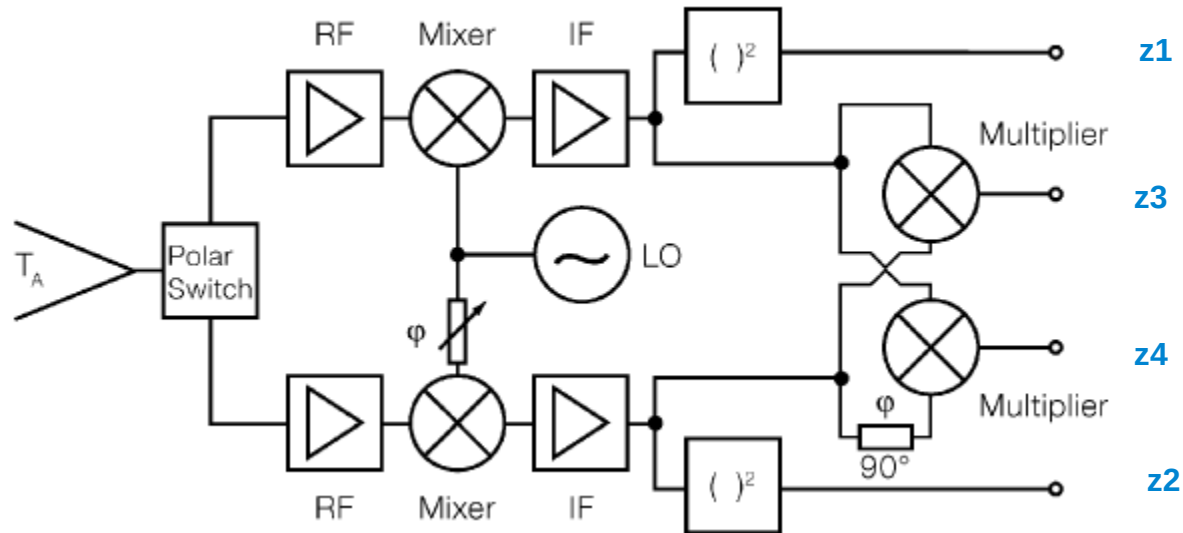
$$S_1 = Ip \cos 2\psi \cos 2\chi$$

$$S_2 = Ip \sin 2\psi \cos 2\chi$$

$$S_3 = Ip \sin 2\chi$$

100% Q	100% U	100% V
<p>+Q</p> <p><math>Q &gt; 0; U = 0; V = 0</math> (a)</p>	<p>+U</p> <p><math>Q = 0; U &gt; 0; V = 0</math> (c)</p>	<p>+V</p> <p><math>Q = 0; U = 0; V &gt; 0</math> (e)</p>
<p>-Q</p> <p><math>Q &lt; 0; U = 0; V = 0</math> (b)</p>	<p>-U</p> <p><math>Q = 0; U &lt; 0; V = 0</math> (d)</p>	<p>-V</p> <p><math>Q = 0; U = 0; V &lt; 0</math> (f)</p>

## Radio polarimetry



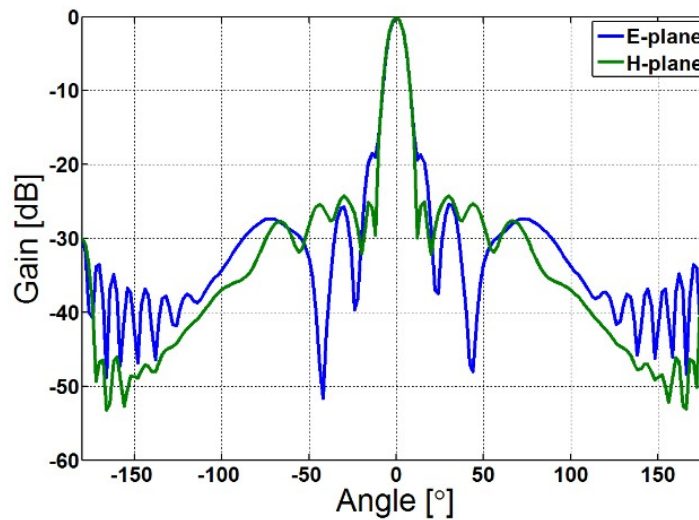
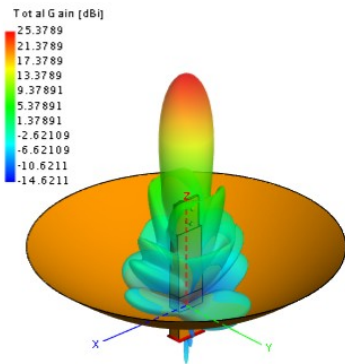
- output  $z_1$  is IF 1 detected by a square-law detector,
- output  $z_2$  is IF 2 detected by a square-law detector,
- output  $z_3$  is the correlation of IF 1 and IF 2,
- output  $z_4$  is the correlation of IF 2 and IF 2 with a phase delay of  $\pi/2$  in one of the channels.

$$\begin{aligned}
 I &= \text{const}(z_1 + z_2), \\
 Q &= \text{const}(z_1 - z_2), \\
 U &= 2 \text{const} z_3, \\
 V &= 2 \text{const} z_4.
 \end{aligned}$$



# Methods of radio astronomy

## Single-dish imaging



Result: Radio map with moderate spatial resolution

Pros: Degraded but real image

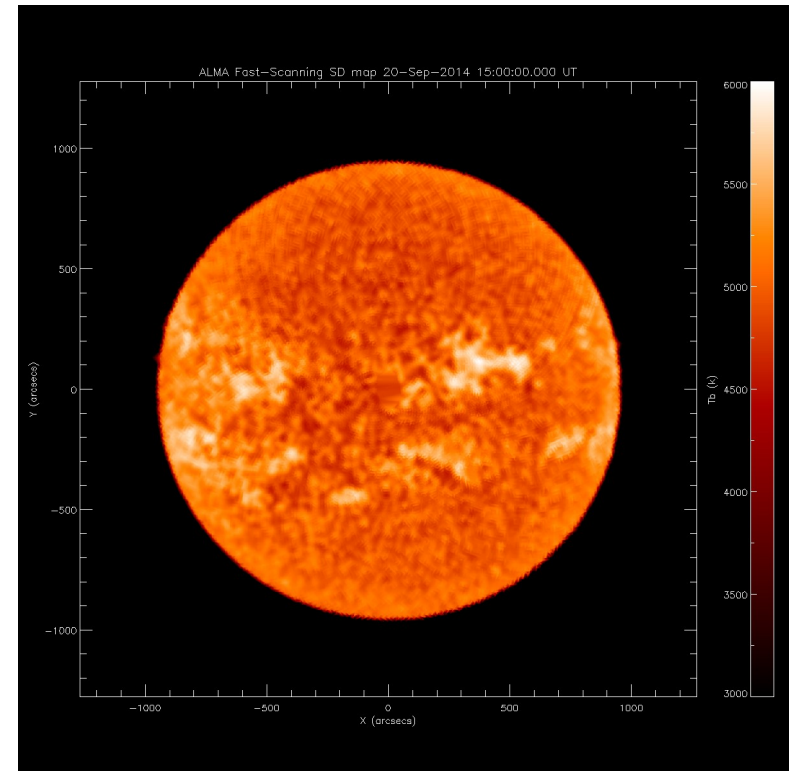
Cons: No spectral information, limited spatial resolution, worse time resolution

# Methods of radio astronomy

## Single-dish imaging

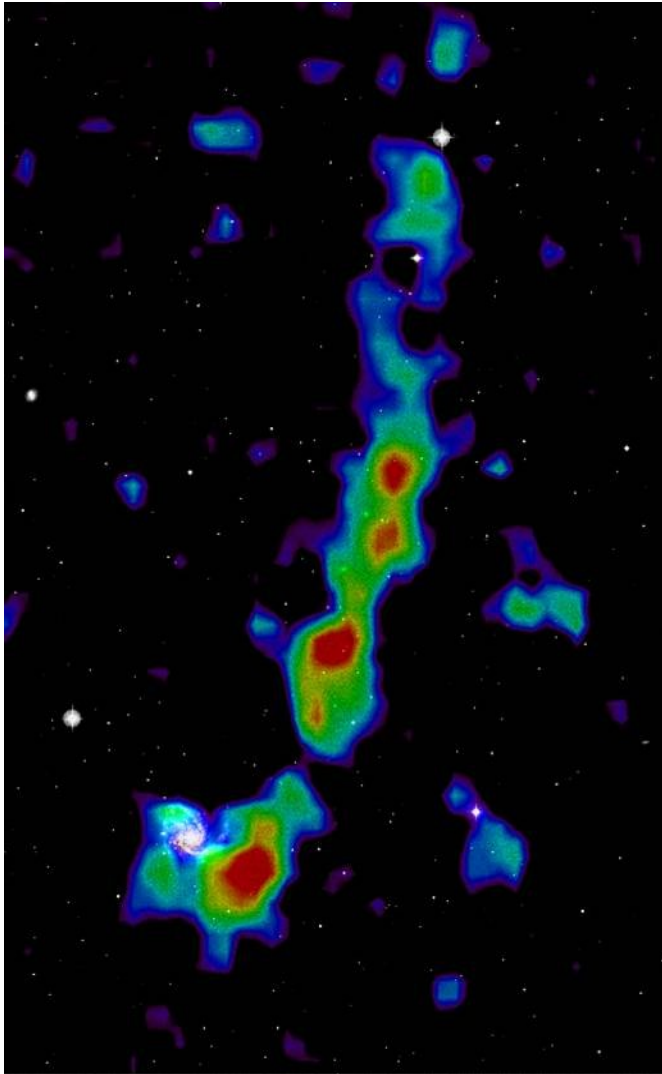


ALMA Fast TP scanning



ALMA single-dish image of the Sun

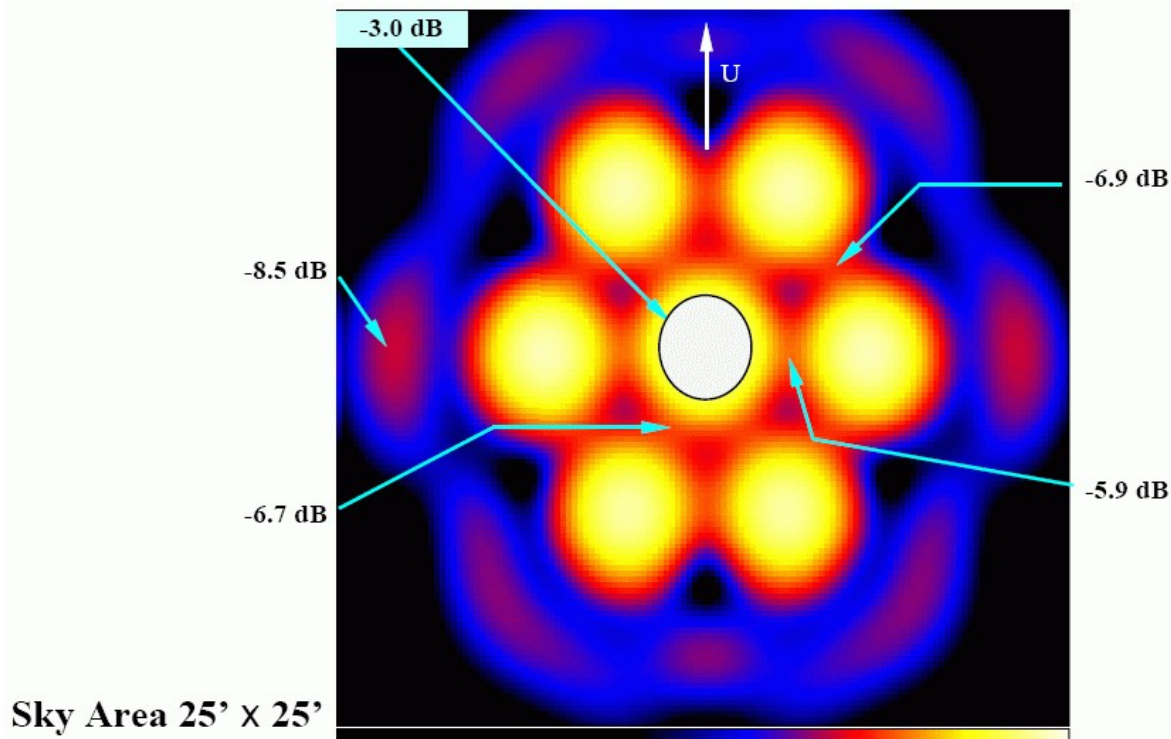
## Single-dish imaging



Gas tail behind a galaxy

# Methods of radio astronomy

## Single-dish imaging



Multi-beam systems: ALFA @ Arecibo



# Trends: Multi-feed arrays – „CCDs“ for radio astronomy

## Single-feed:

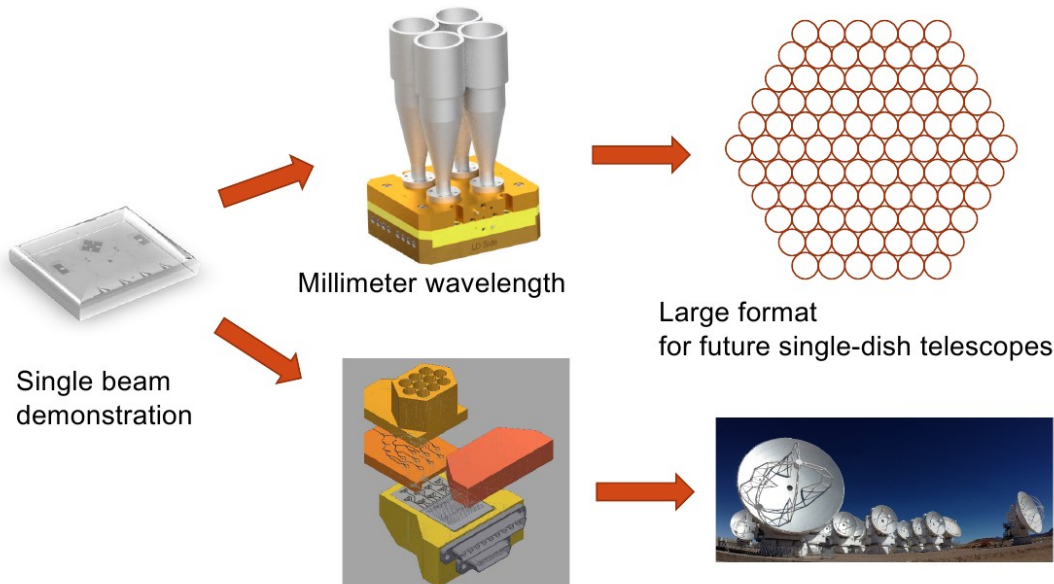
FoV given by a **primary beam** of a single antenna (ALMA 12m:  $\sim 1'$ @100GHz). For larger fields we use mosaicing (i.e. consecutive re-pointing).

## Multi-feed:

Larger FoV, faster mosaicing/scanning



ALFA@Arecibo



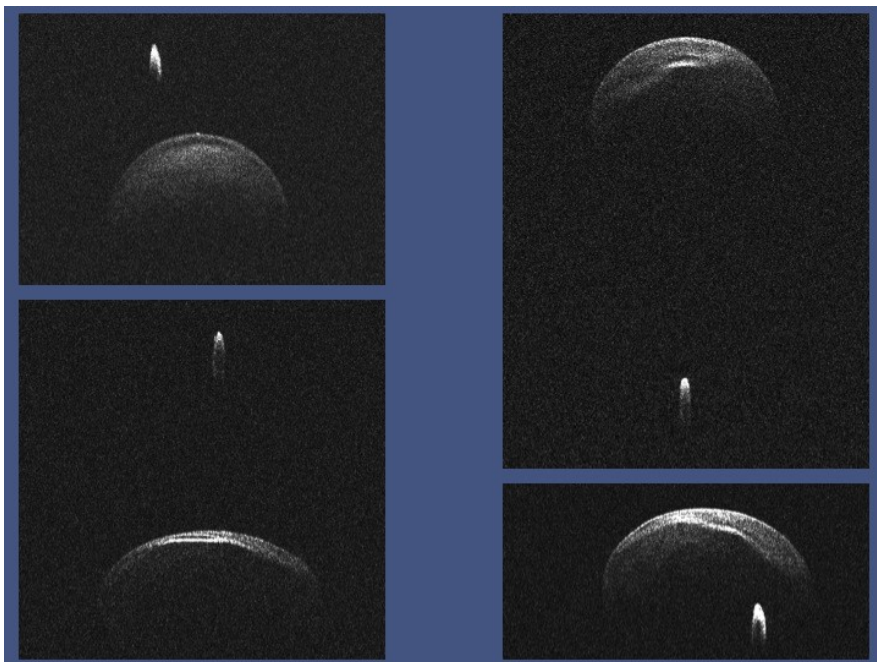
Phased multi-array@ASKAP

Applications nowadays even in mm/sub-mm range

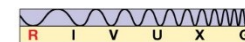
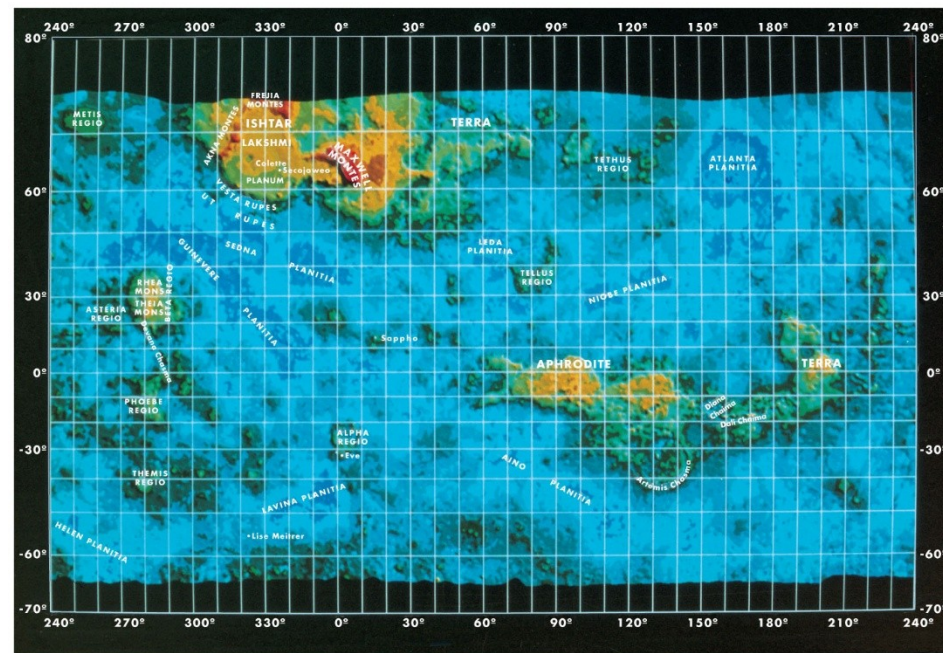
(see ALMA Development Workshop 2019 [on-line proceedings](#))

# Methods of radio astronomy

## Active radio astronomy: Radar sounding



Binary asteroid KW4 by Arecibo



Venus surface (Magellan)

### Radar Equation

We begin by reviewing the basic monostatic radar range equation describing received power for a radar system:

$$P_r = \frac{P_t G_t G_r \lambda^2 PF^4 \sigma_{RCS}}{(4\pi)^3 r^4 L}$$

Where  $P_t$  = Transmitted power  
 $G_t$  = Transmit antenna gain  
 $G_r$  = Receive antenna gain  
 $\lambda$  = Radar wavelength  
 $PF$  = Pattern Propagation Factor  
 $r$  = Slant range from radar to target  
 $\sigma_{RCS}$  = Target radar cross section (RCS)  
 $L$  = Miscellaneous system losses

$$R_{\max} = \sqrt[4]{\frac{P_S \cdot G^2 \cdot \lambda^2 \cdot \sigma}{P_{E_{\min}} \cdot (4\pi)^3 \cdot L_{ges}}}$$



# Methods of radio astronomy

## Active radio astronomy: Radar sounding



Arecibo – had an active radar

FAST:  
just a passive radio telescope



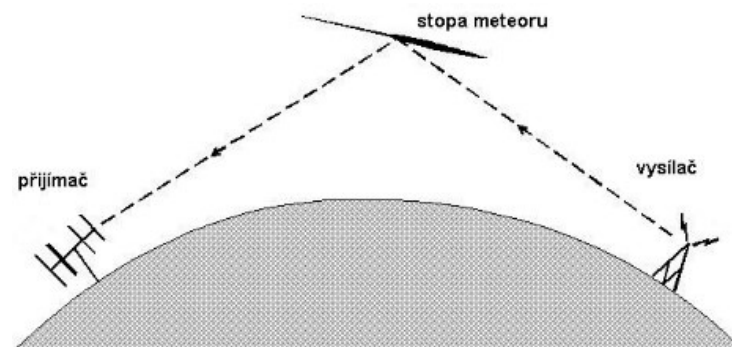


# Methods of radio astronomy

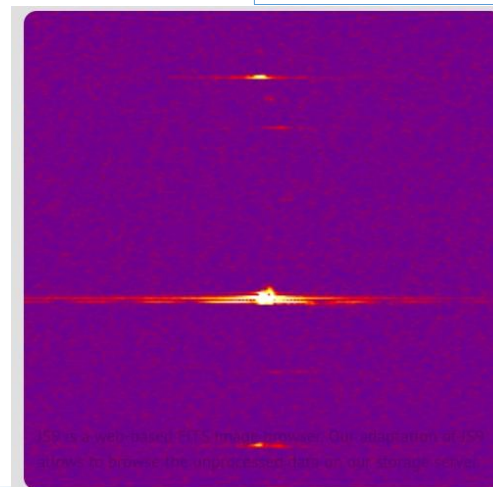
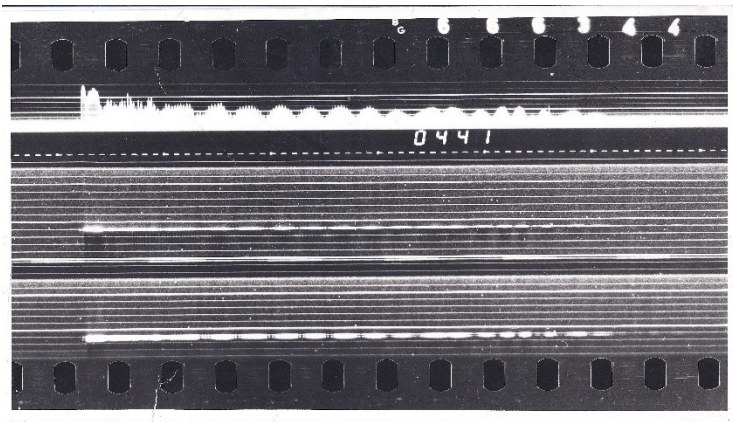
## Active radio astronomy: Radar sounding



## Radar-based meteor research



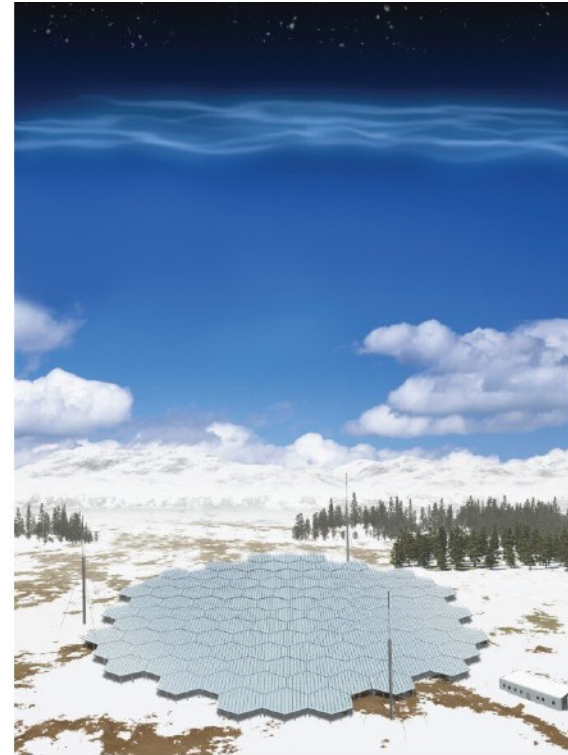
Amateur radio observations of meteors  
e.g. <http://bolidozor.cz>



# Methods of radio astronomy

Active radio astronomy: Radar sounding

EISCAT3D: Ionospheric sounding + space debris search



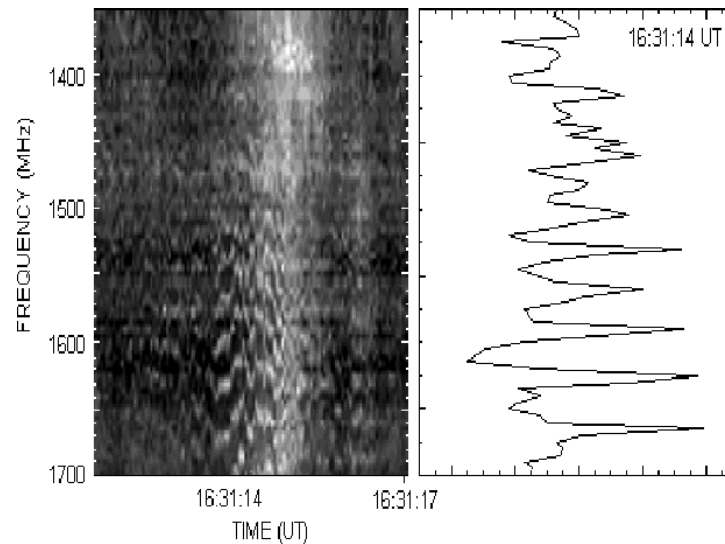
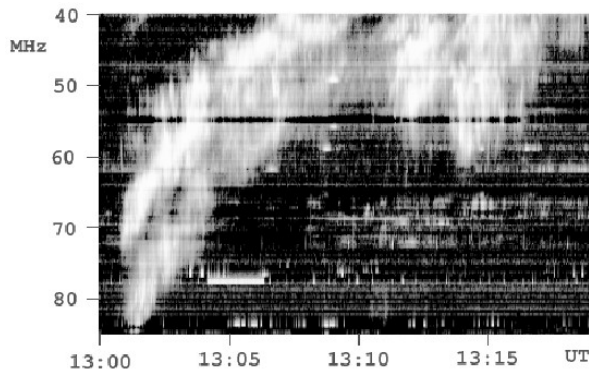
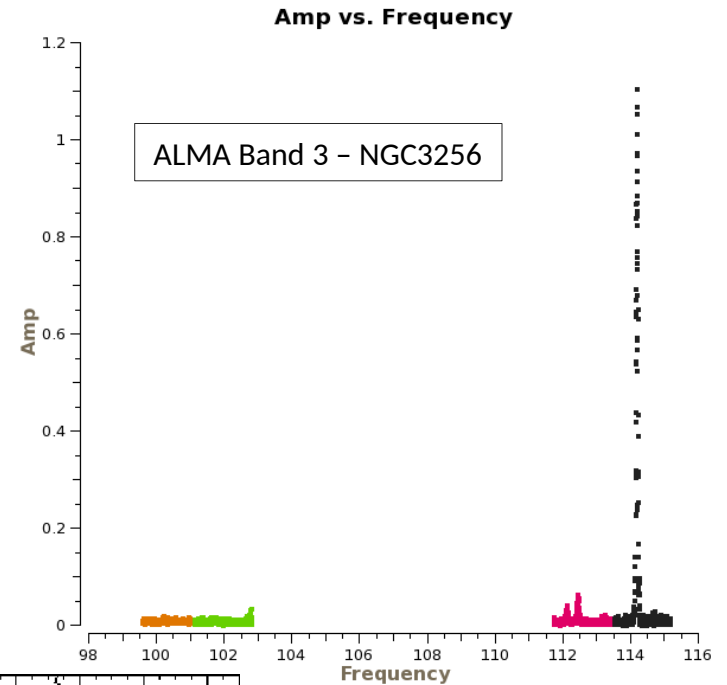
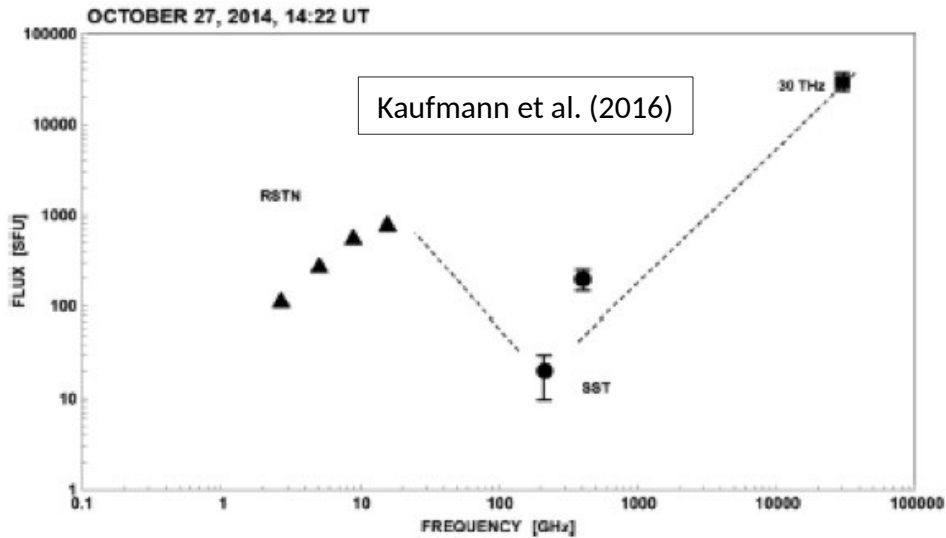
Phased-array radar

Phased radar GIF



# Methods of radio astronomy

## (single-dish) Radio spectroscopy



Result: (dynamic) spektra

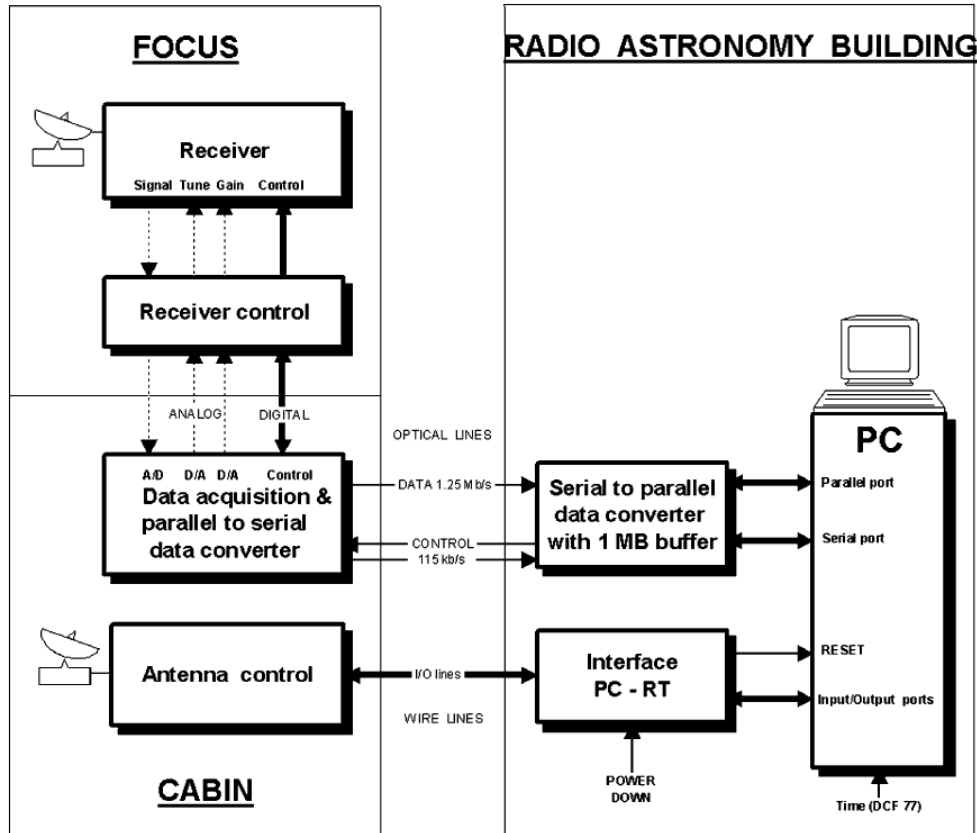
Pros: (rather) Good time resolution full spectra

Cons: No spatial resolution

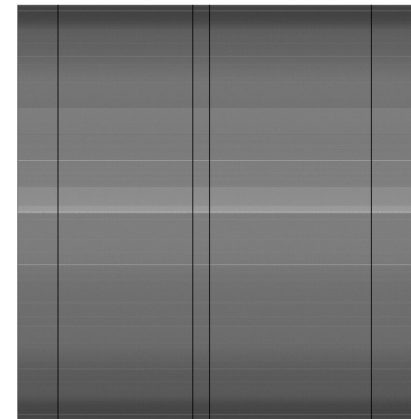


# Methods of radio astronomy

## (single-dish) Radio spectroscopy



RT5 @ Ondrejov: Sweeping spectrograph

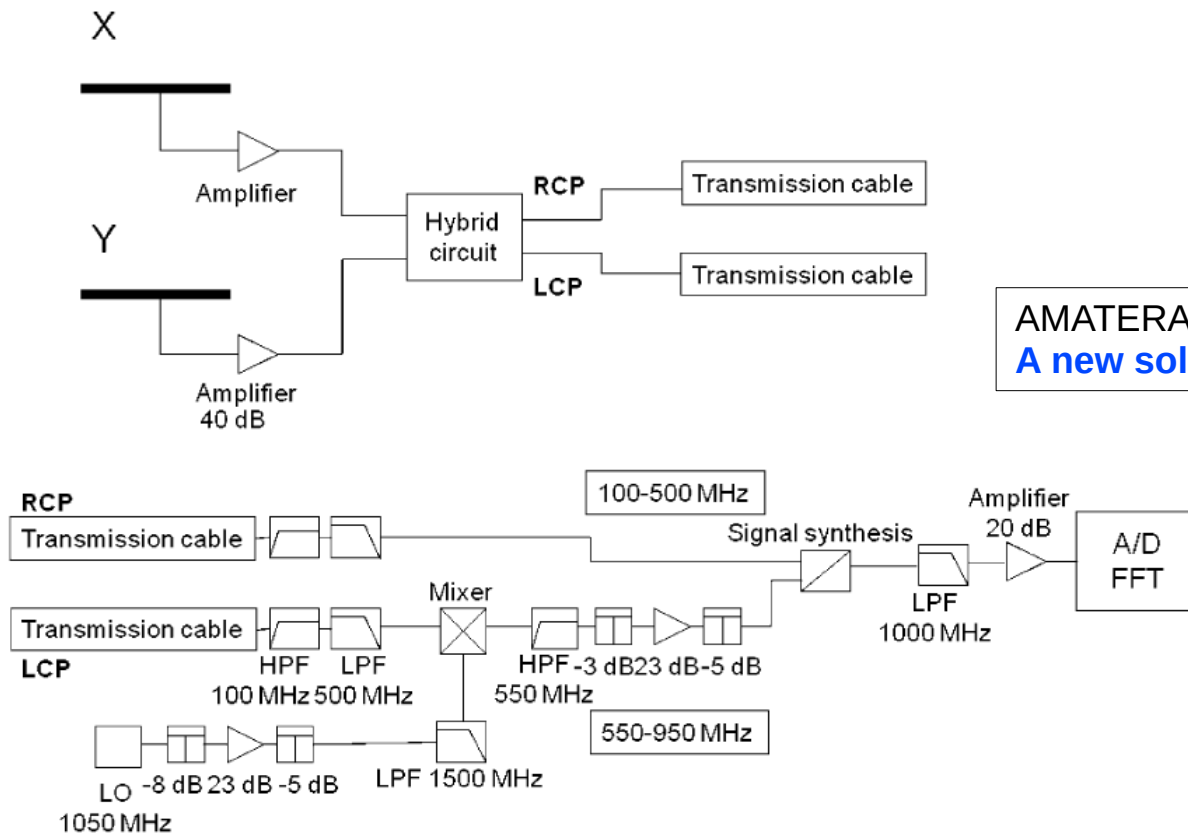


$\Delta f = 250 \text{ kHz}$   
 $\Delta t = 1 \text{ ms}$

RT5 @ Ondrejov: New tested Fourier-based (autocorr) spectrograph - 1<sup>st</sup> results

# Methods of radio astronomy

## (single-dish) Radio spectroscopy/spectro-polarimetry

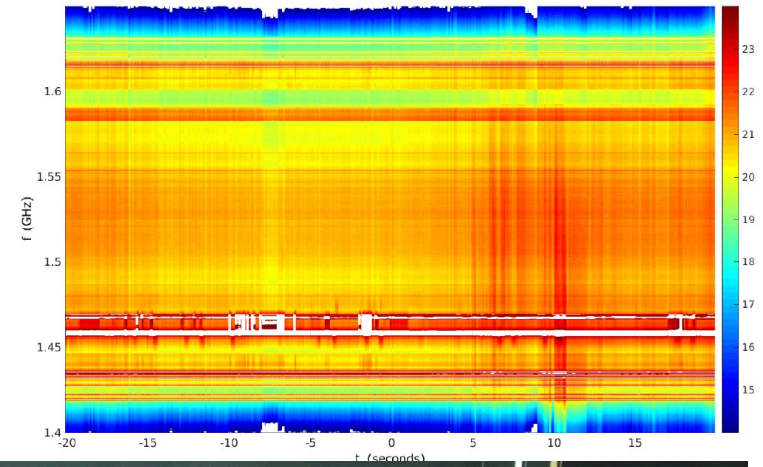


AMATERAS/IPRT Nobeyama, Japan  
[A new solar radio spectrorgraph @ Ondrejov](#)

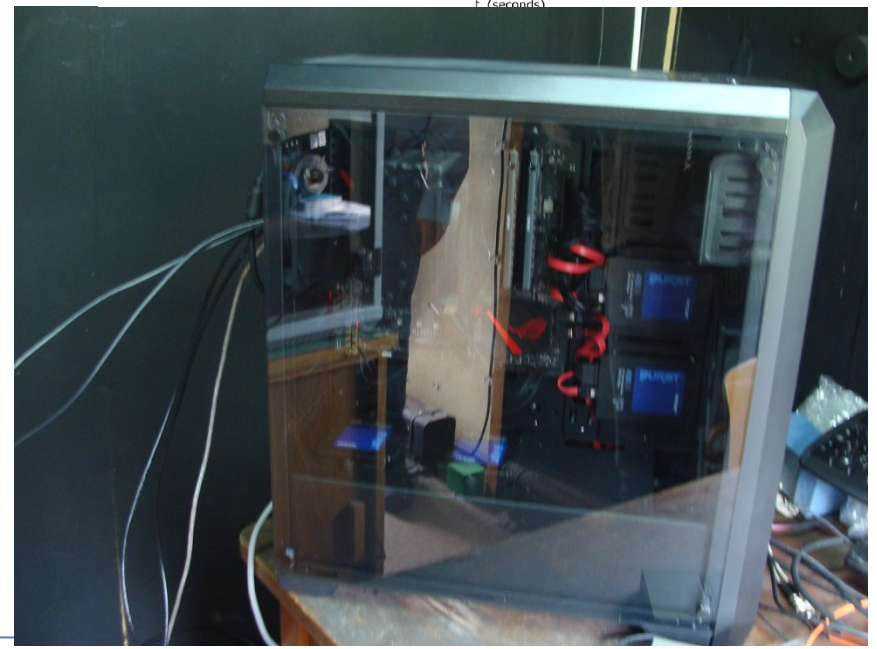
Figure 2 Block diagram of IPRT/AMATERAS.

# DSP technology now affordable for small & middle-size observatories

Spectrographs/spectro-polarimeters built as Software-Defined Radio  
Ondrejov Solar hi-Cadence Automated Radio Spectrograph(s) / OSCARS

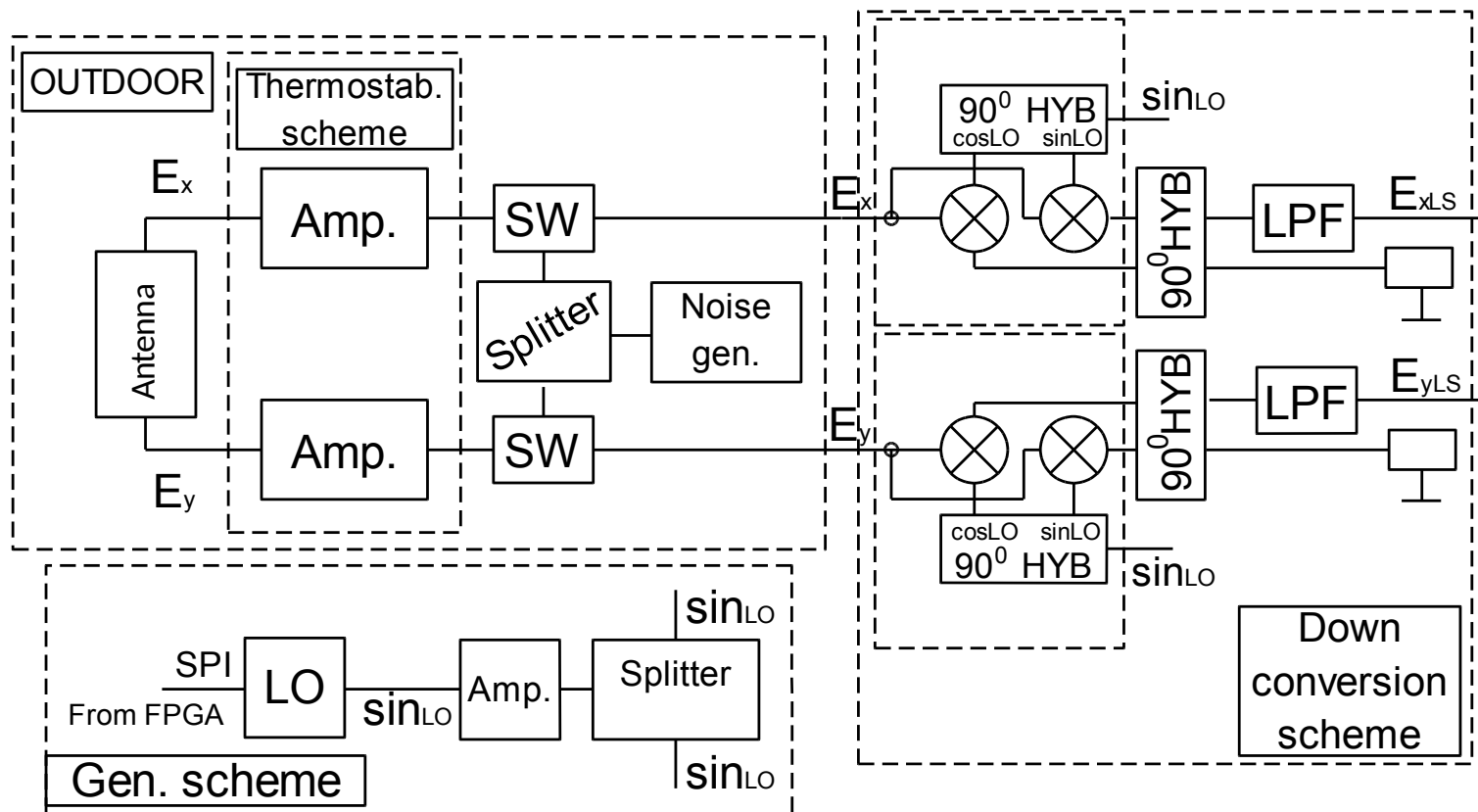


(Puričer, Kovář & Bárta, [Electronics 2019](#))



# Methods of radio astronomy

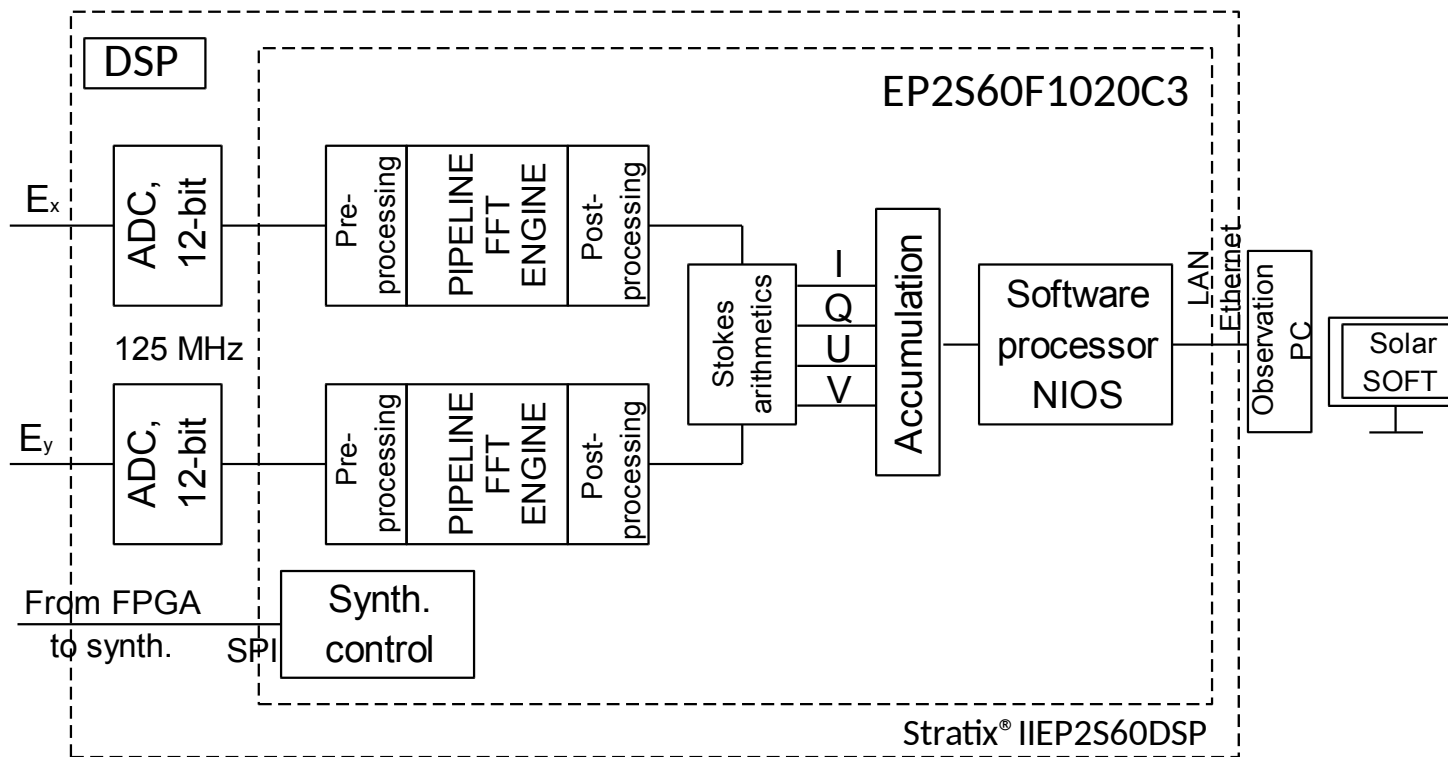
## Radio spectro-polarimetry



SSPM Badary/Irkutsk, Russia

# Methods of radio astronomy

## Radio spectro-polarimetry

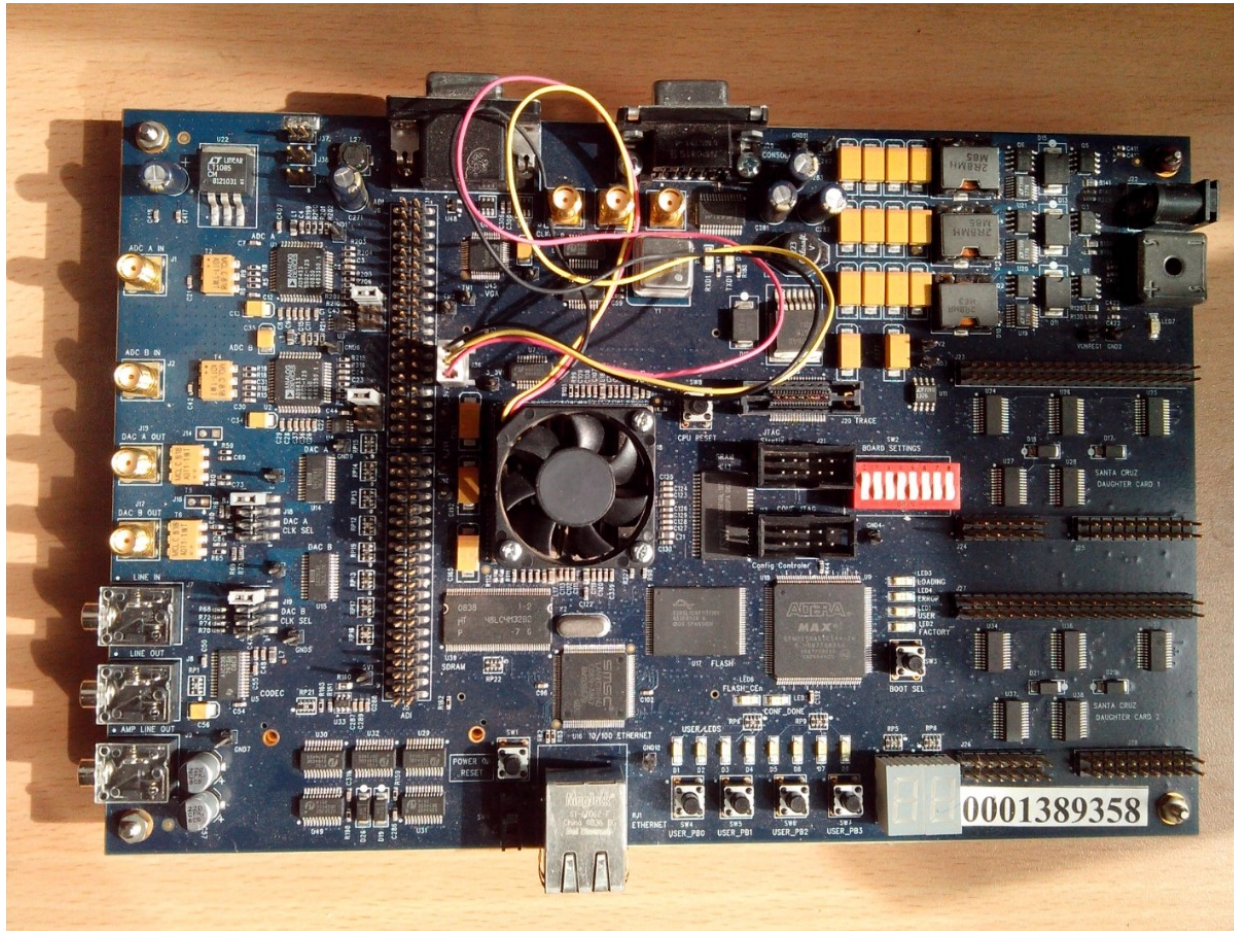


SSPM Badary/Irkutsk, Russia



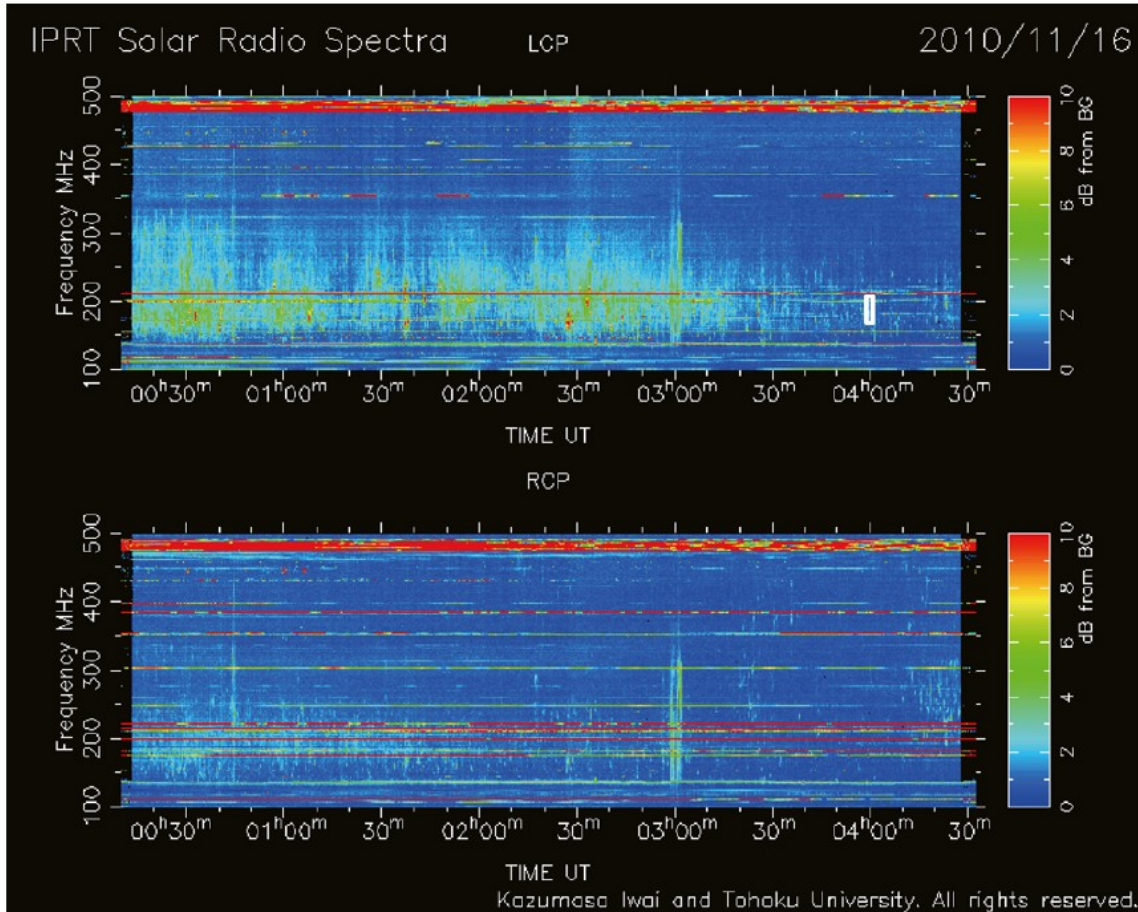
# Methods of radio astronomy

## Radiospectro-polarimetry



SSPM Badary/Irkutsk, Russia

## Radiospectro-polarimetry



Result: (dynamic) spektra

Pros: (rather) Good time resolution, full spectra, **polarisation**

Cons: No spatial resolution

## Radio interferometry

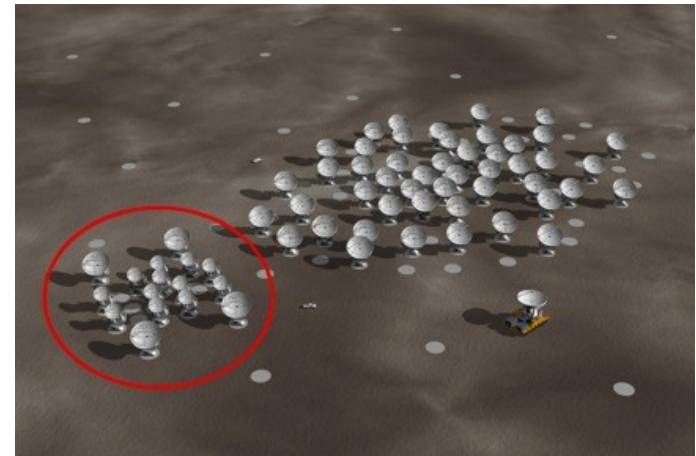
### Phased arrays – sparse rows

$$|I_{\Sigma}(\theta)| = \frac{1 - \cos N \frac{2\pi D}{\lambda} \theta}{1 - \cos \frac{2\pi D}{\lambda} \theta}$$



### Aperture synthesis – baseline signal correlations

$$\langle I_{AB} \rangle(\theta) = \int_{-\infty}^{+\infty} B(\theta) \exp\left(i \frac{2\pi D}{\lambda} \theta\right) d\theta$$

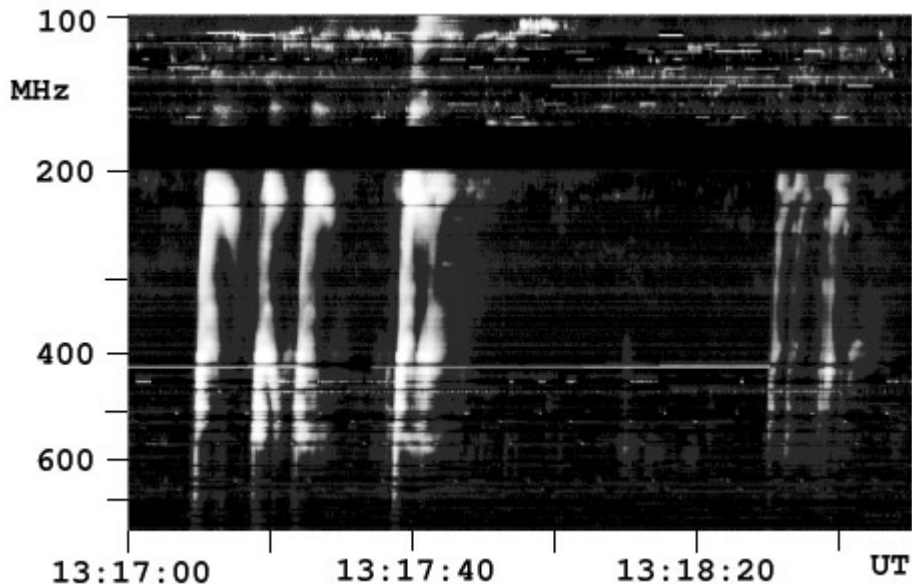


# Radio spectroscopy in closer view

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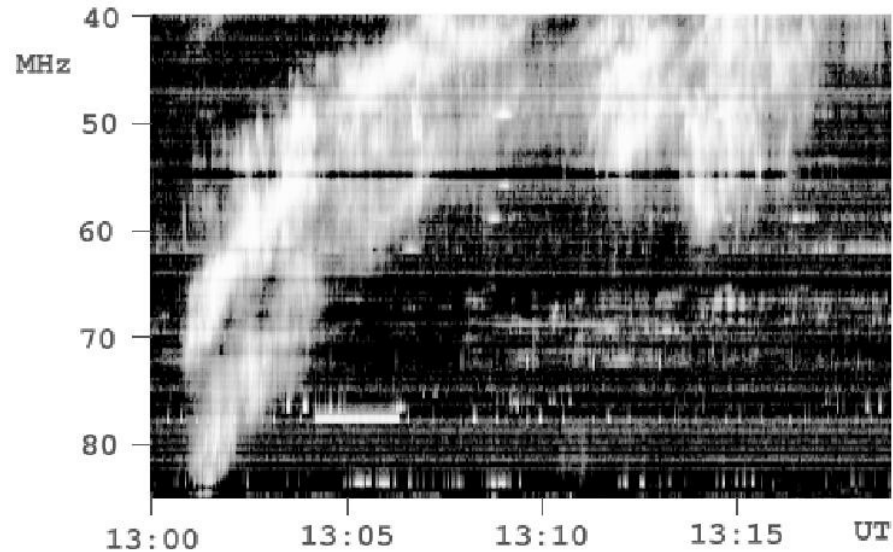
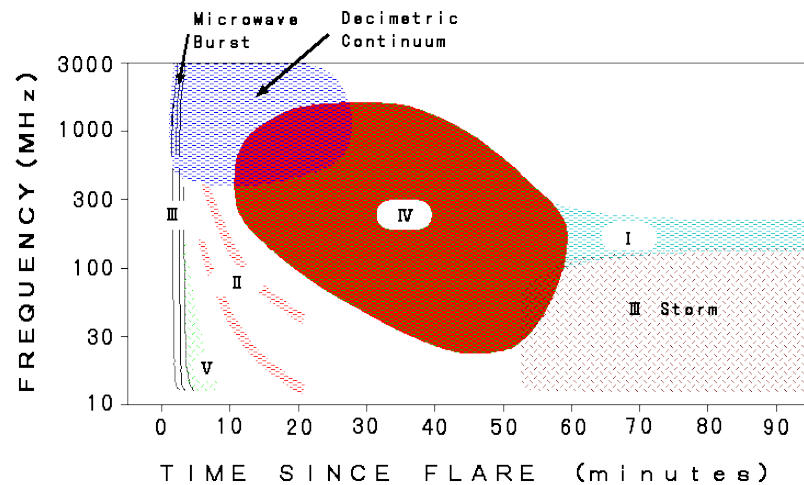


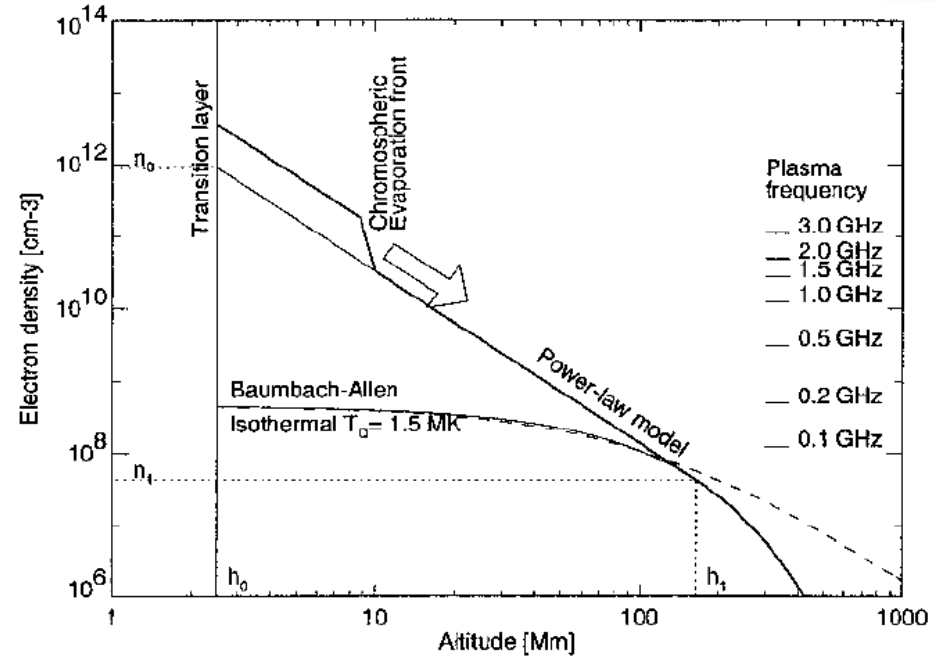
## Solar radio dynamic spectroscopy



$$\omega_{pe} = \sqrt{\frac{n_e e^2}{m^* \epsilon_0}}$$

$$f_{pe} \approx 8980 \sqrt{n_e} \text{ Hz}$$





$$\omega_{pe} = \sqrt{\frac{n_e e^2}{m^* \epsilon_0}}$$

$$f_{pe} \approx 8980 \sqrt{n_e} \text{ Hz}$$

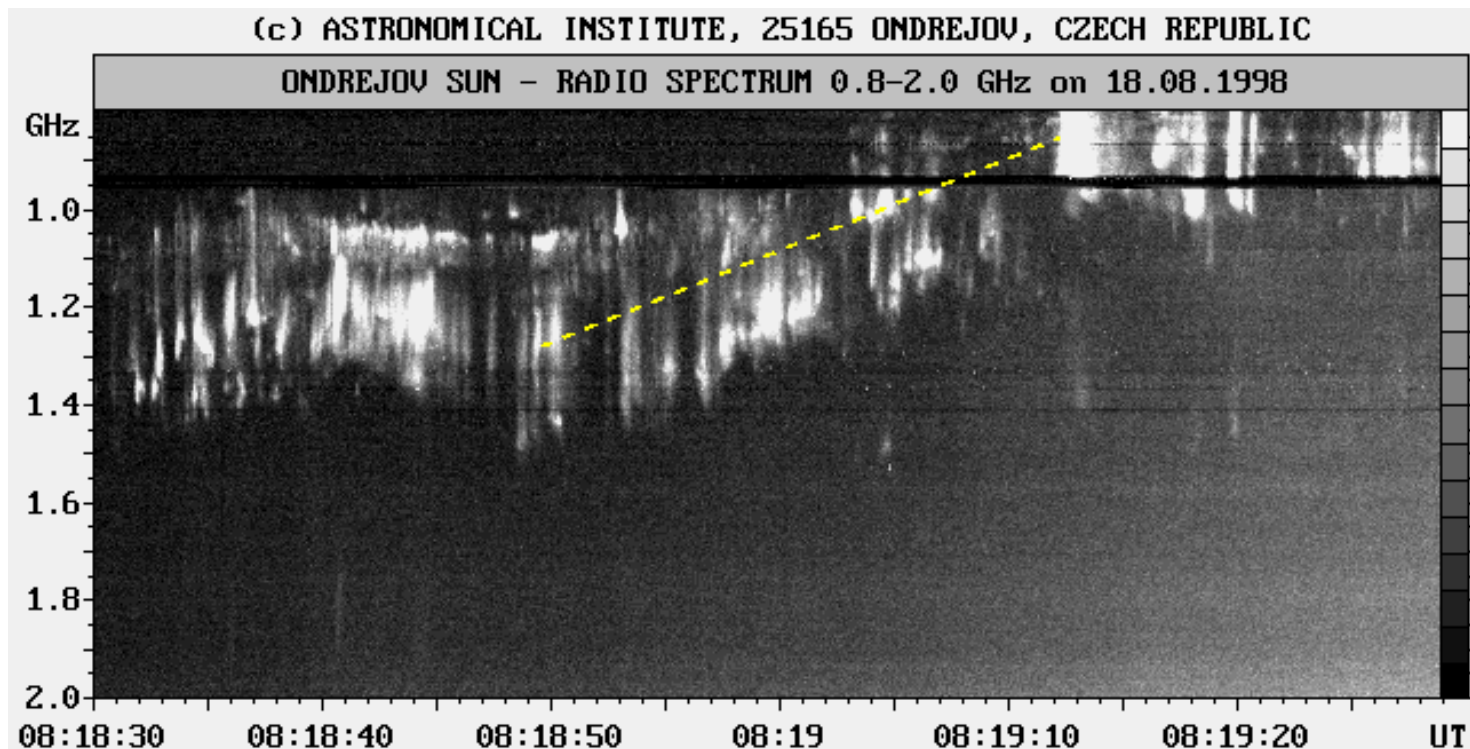
$$\frac{\Delta f}{\Delta t} = \frac{\Delta f}{\Delta n} \cdot \frac{\Delta n}{\Delta h} \cdot v$$

$$n(h) = n_1 \left( \frac{h}{h_1} \right)^{-p}$$

$$v = 4.55 \times 10^{14} f^{-1.84} \frac{df}{dt}$$

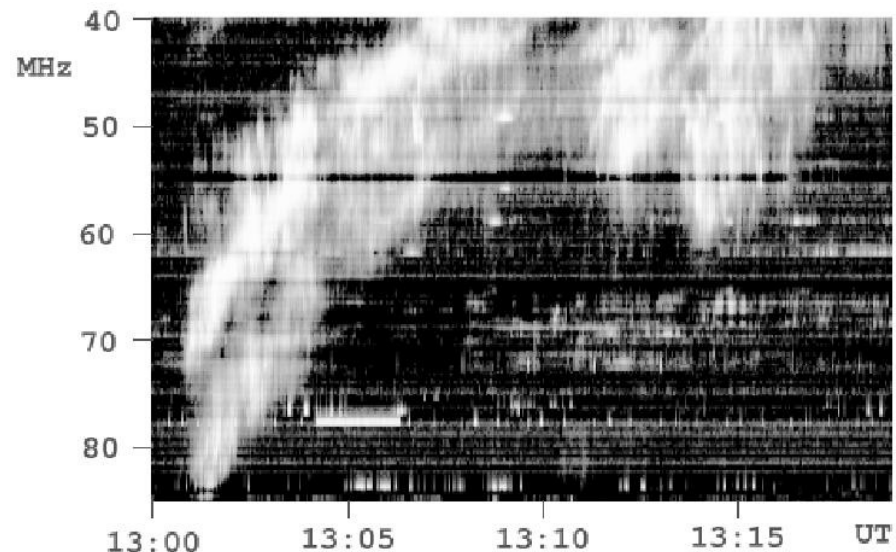
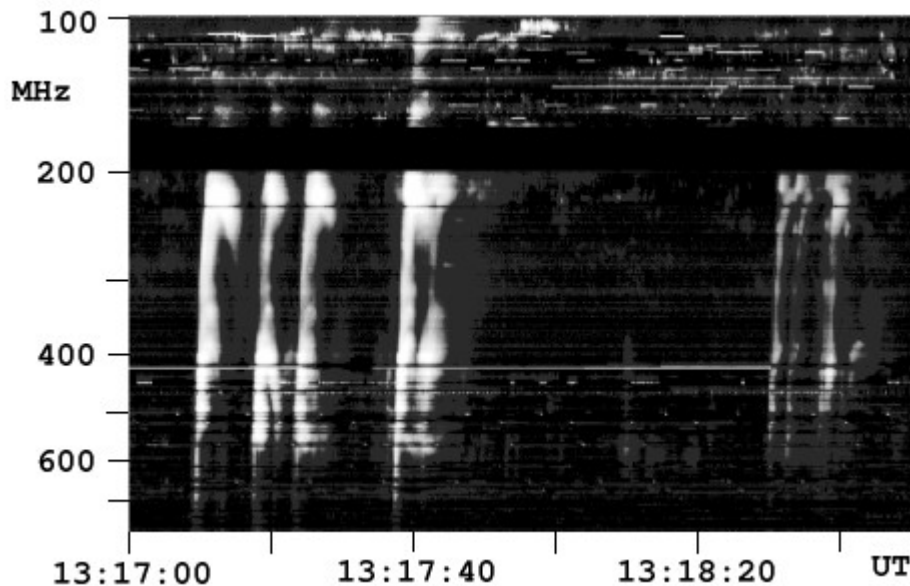
# Methods of radio astronomy

(Solar) radio dynamic spectroscopy



$$\frac{df}{dt} = \frac{f_2 - f_1}{t_2 - t_1}$$

Frequency drift



$$\omega_{pe} = \sqrt{\frac{n_e e^2}{m^* \epsilon_0}}$$

$$f_{pe} \approx 8980 \sqrt{n_e} \text{ Hz}$$

$$\frac{\Delta f}{\Delta t} = \frac{\Delta f}{\Delta n} \cdot \frac{\Delta n}{\Delta h} \cdot v$$

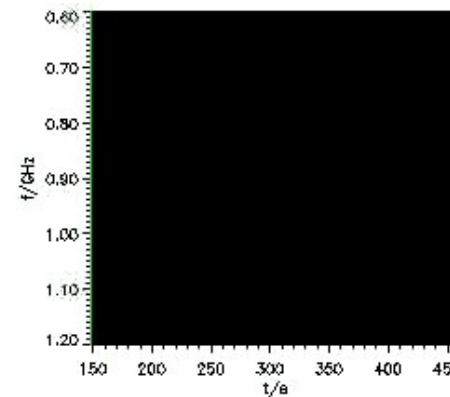
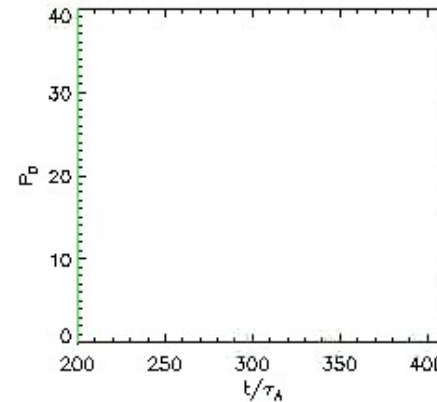
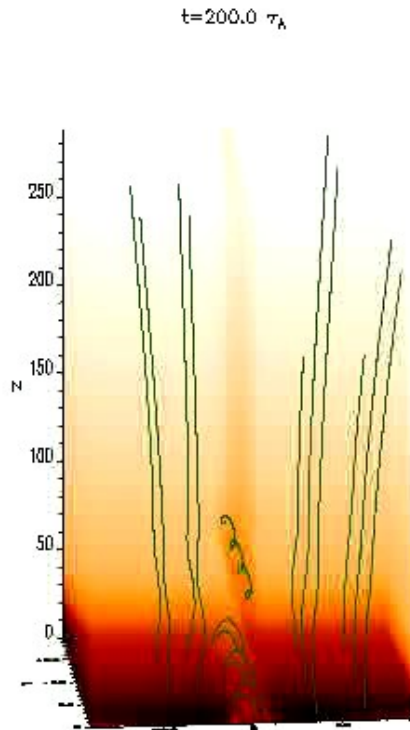
$$n(h) = n_1 \left( \frac{h}{h_1} \right)^{-p}$$

$$v = 4.55 \times 10^{14} f^{-1.84} \frac{df}{dt}$$



# Plasmoid ejecta & DPS radio bursts

Karlicky 2004, Barta et al., 2008

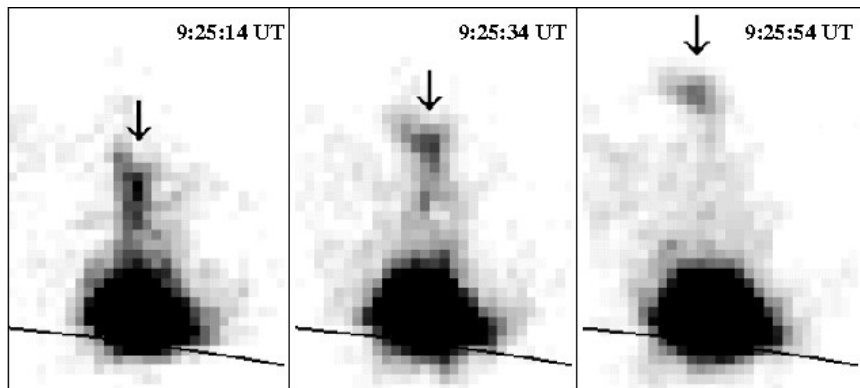


Bárta & Karlický, 2007

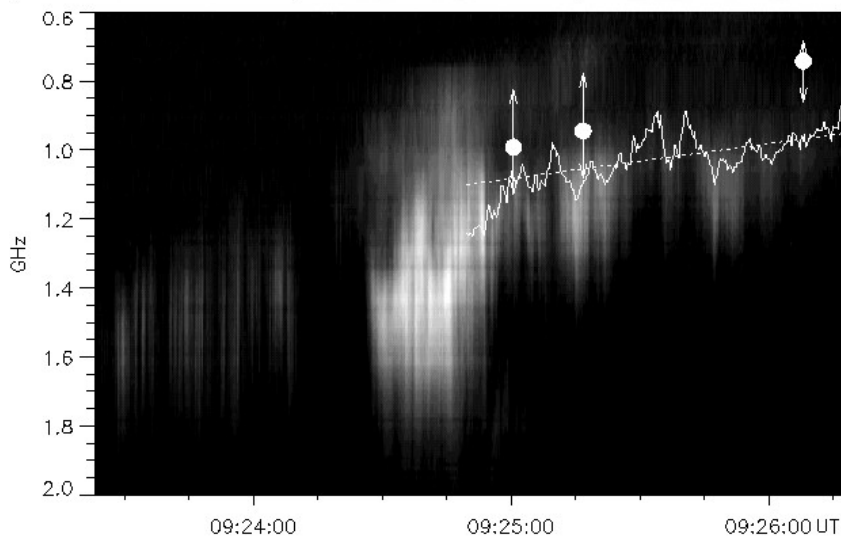


Model: DPS radio spectrum formation

### Plasmoid ejecta & DPS radio bursts



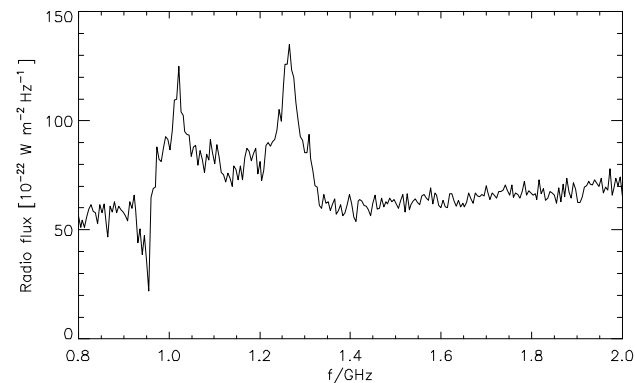
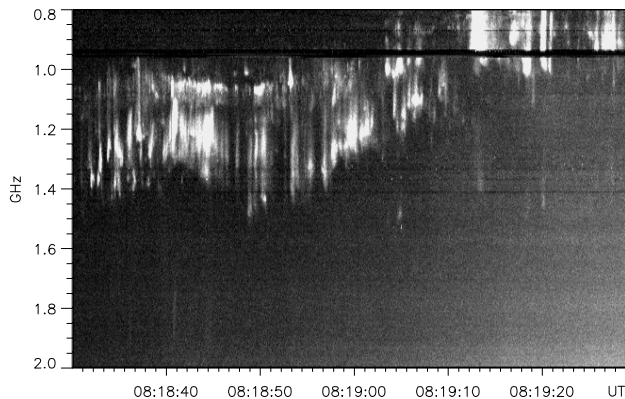
Ohyama & Shibata, 1998, ApJ 499



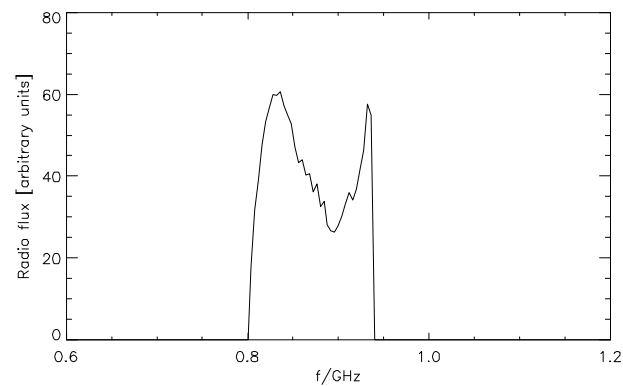
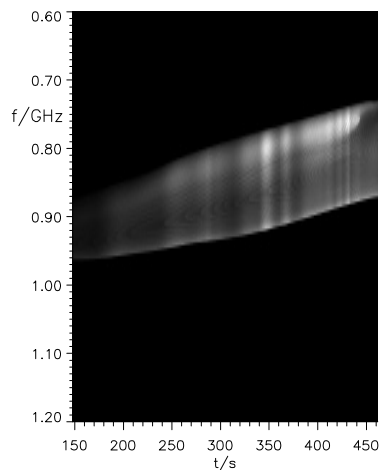
Kliem, Karlicky & Benz, 2000, A&A 360

# Plasmoid ejecta & DPS radio bursts

Observation



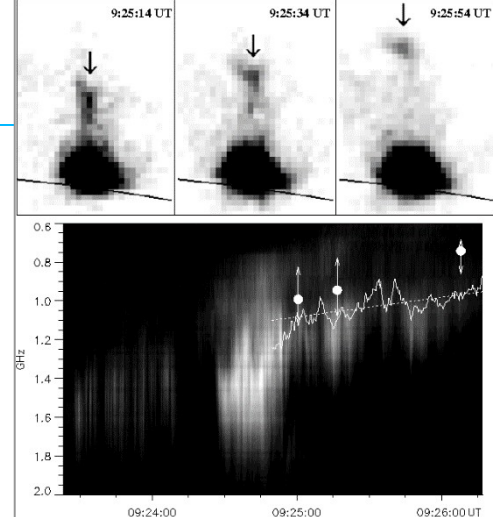
Model



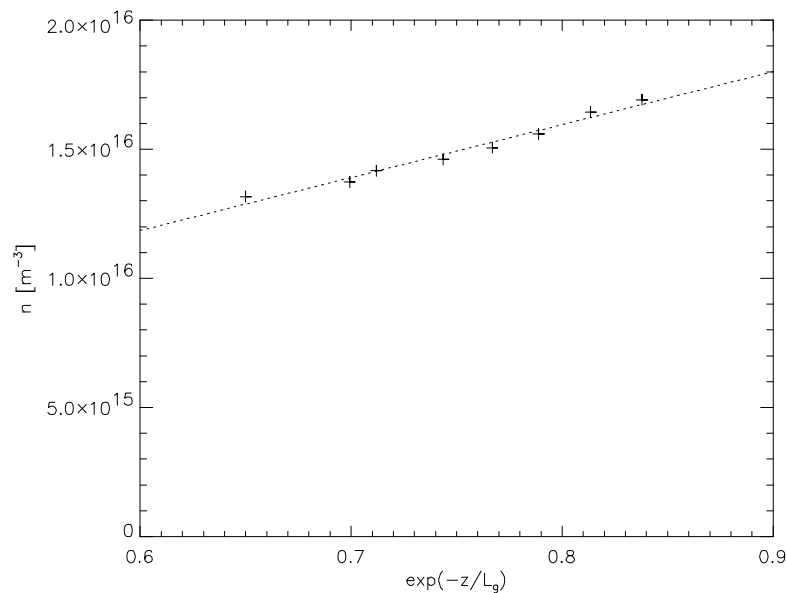
Relations to observation

Barta, Karlicky & Zemlicka 2008, SPh

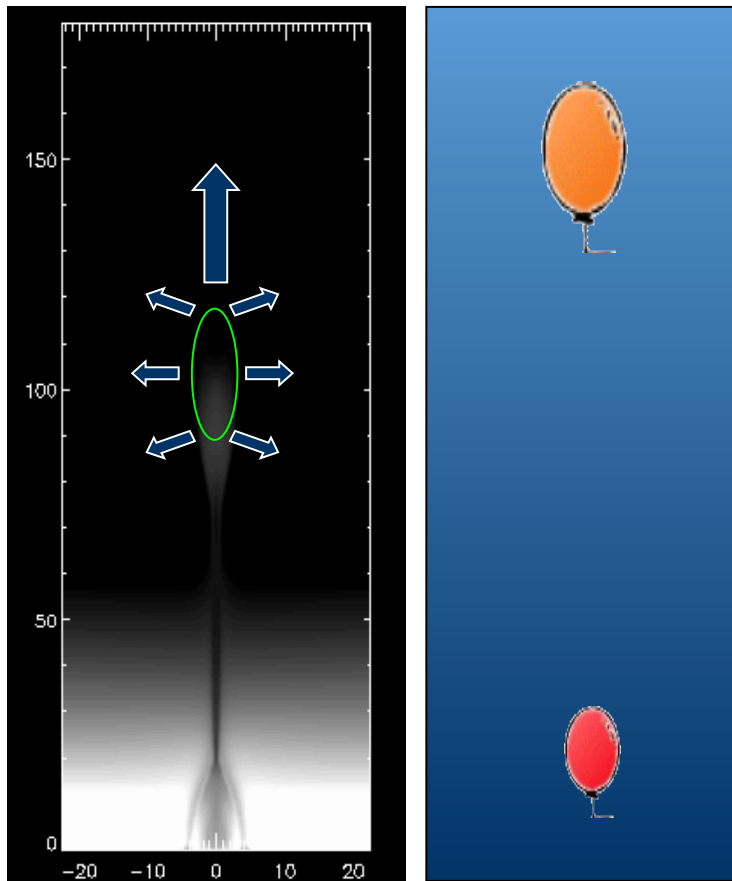
### Plasmoid ejecta & DPS radio bursts



### Model-based diagnostics

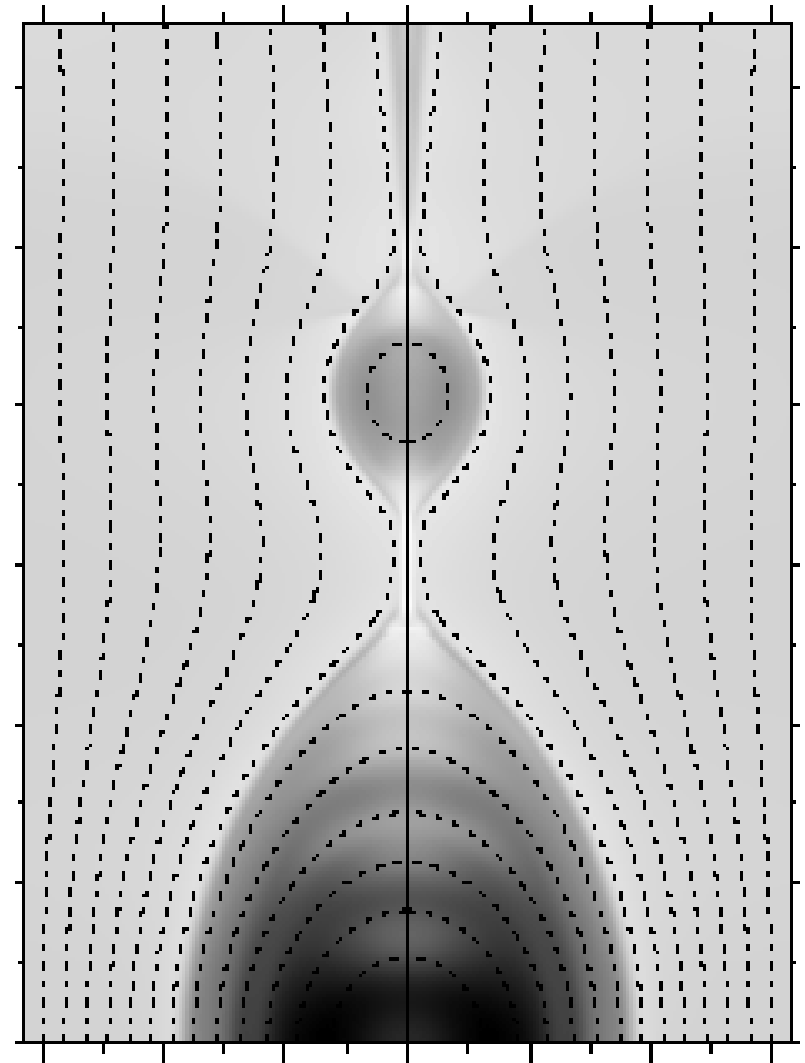
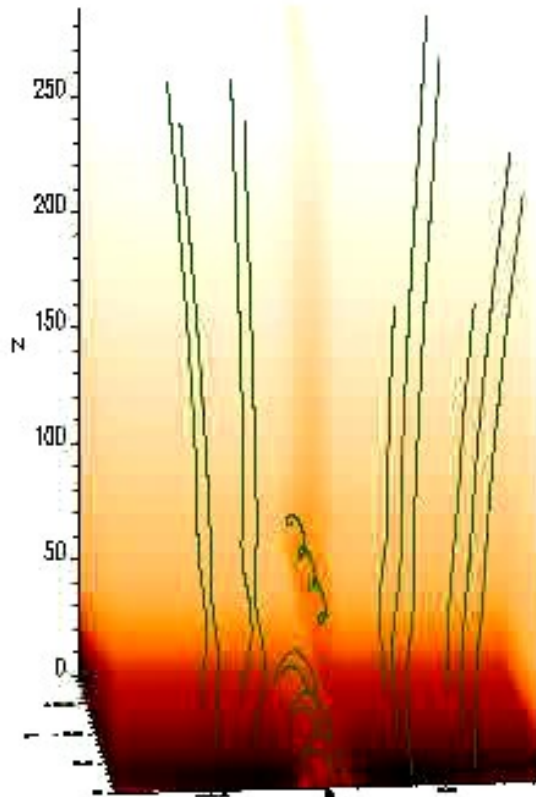


$$n_{in} = \frac{p_0}{2k_B T_{in}} \cdot \exp\left(-\frac{z}{L_g}\right) + \frac{B_{out}^2}{4\mu_0 k_B T_{in}}$$



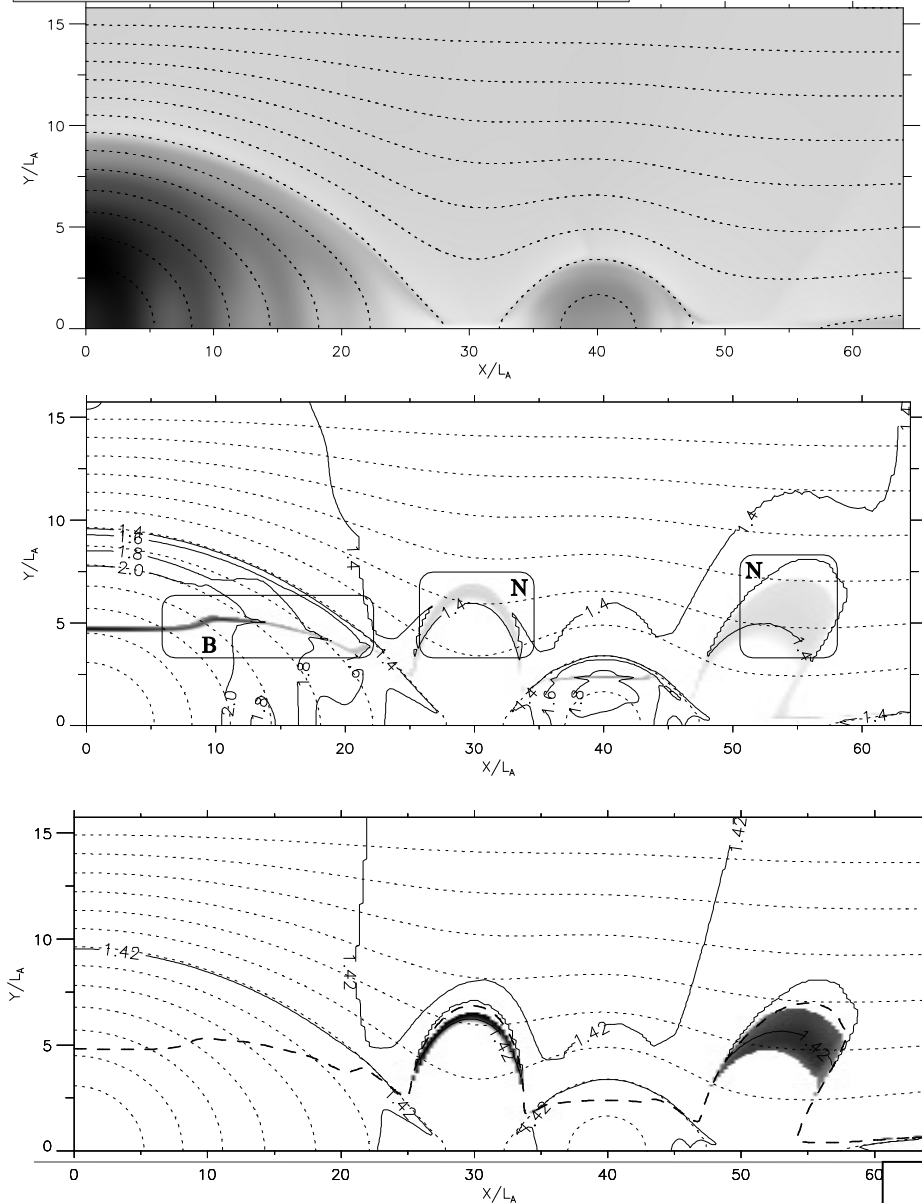
### Radio signatures of turbulence in the reconnection outflows

$t=200.0 \tau_A$

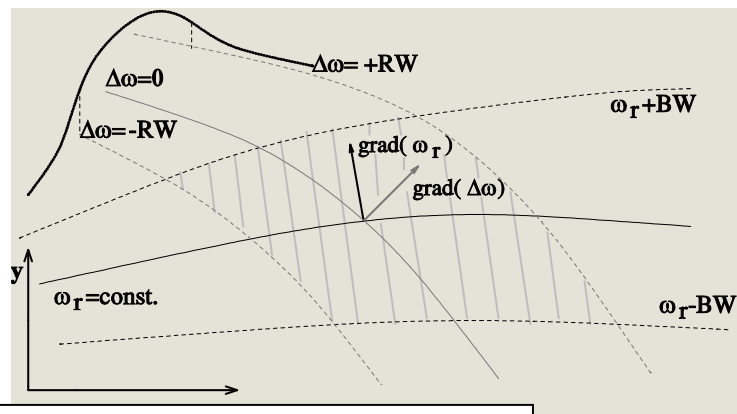
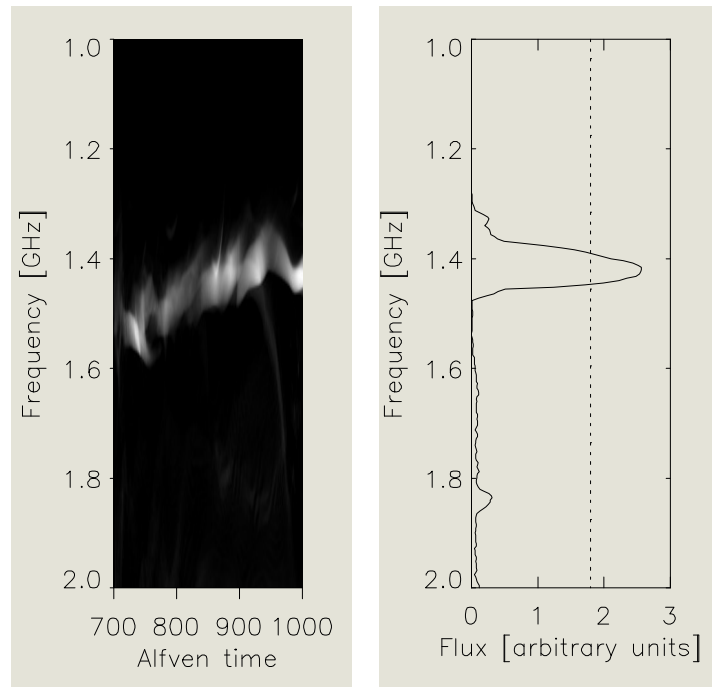


# Methods of radio astronomy

## Solar radio dynamic spectroscopy



## Radio signatures of turbulence in the reconnection outflows

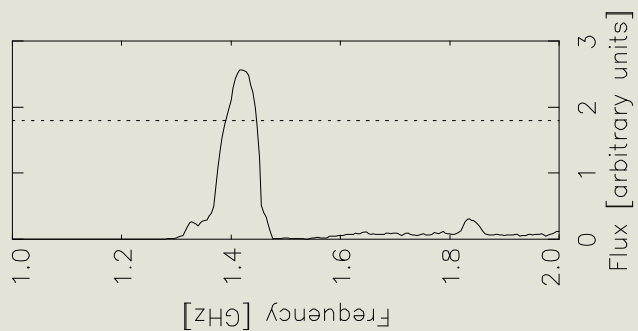
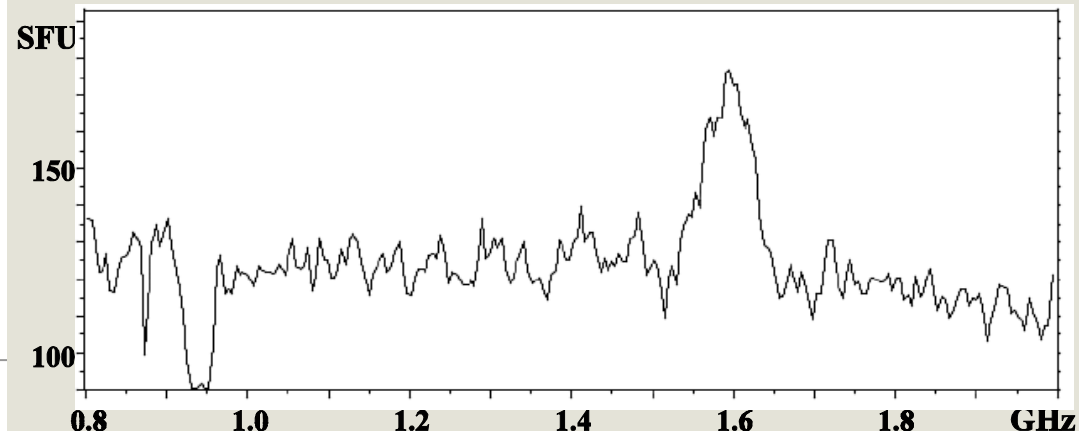
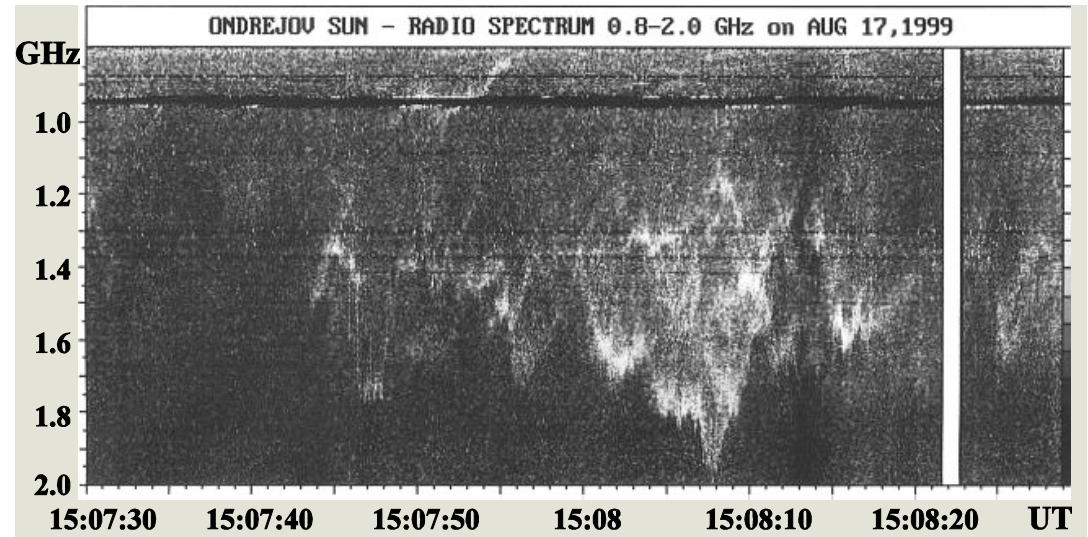
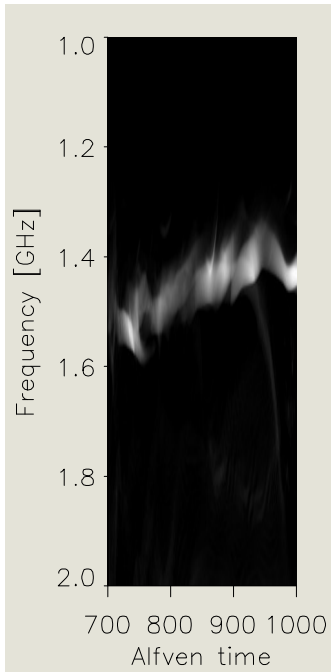


# Methods of radio astronomy

Solar radio dynamic spectroscopy

Radio signatures of turbulence in the reconnection outflows

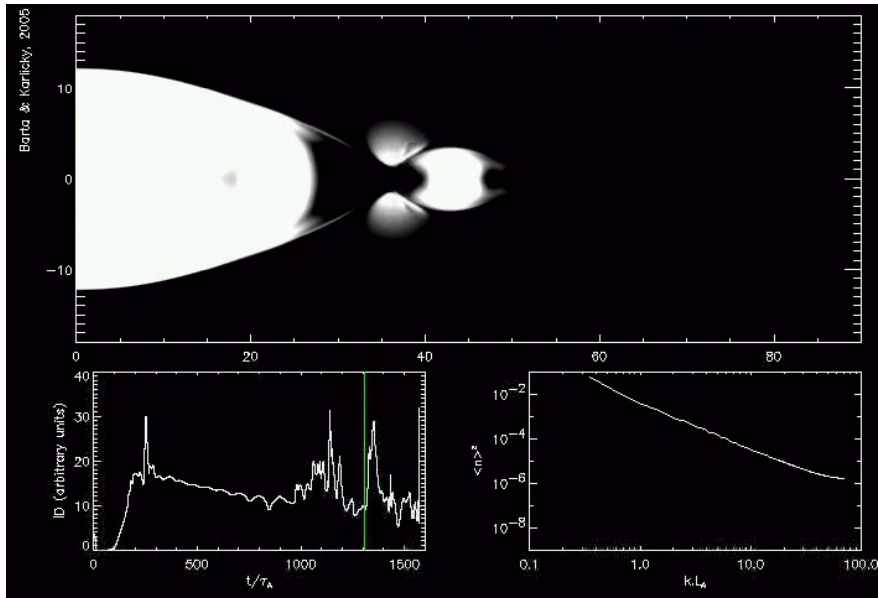
Lace burst



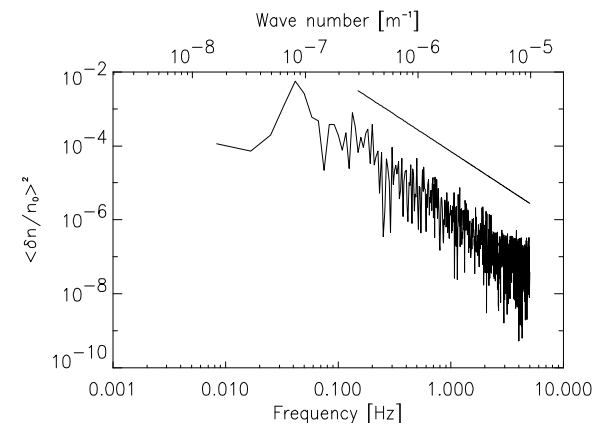
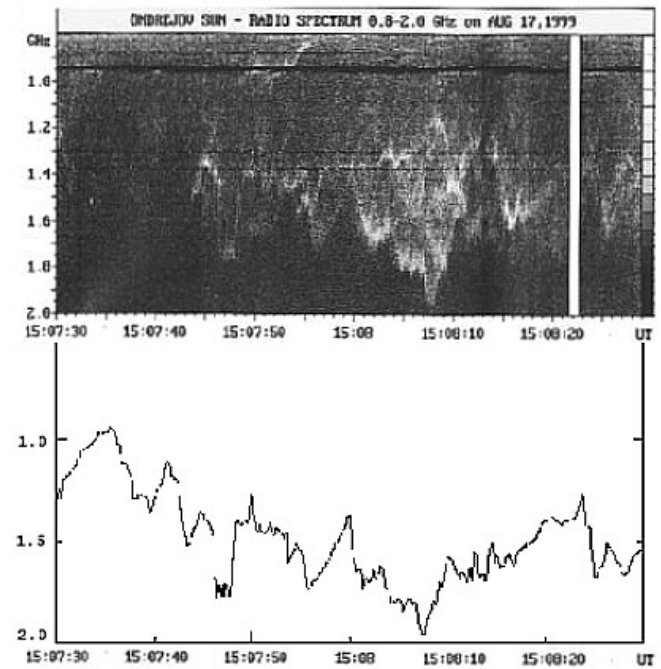


# Methods of radio astronomy

## Solar radio dynamic spectroscopy

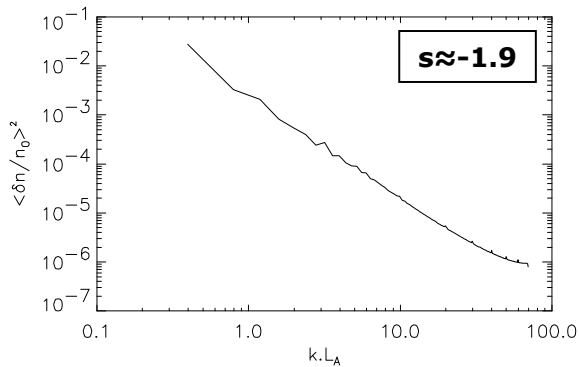


Radio signatures of turbulence  
in the reconnection outflows

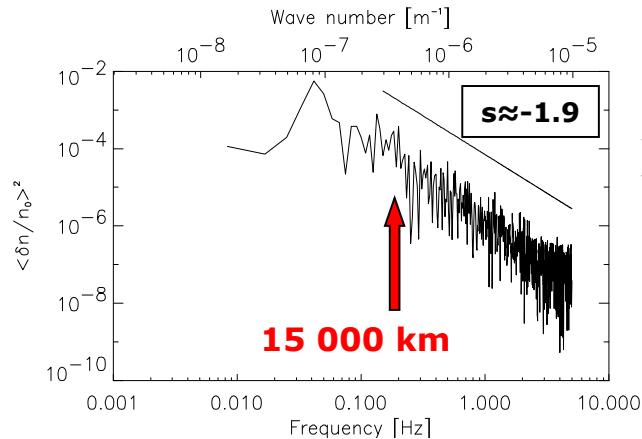




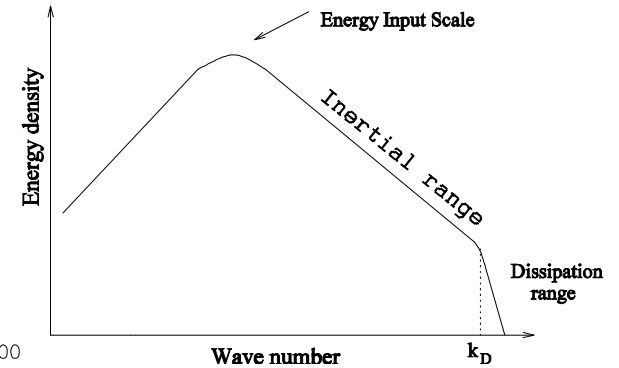
### Density fluctuations



Model



Inferred from observations  
(lace burst)



HD turbulence  
(scheme)

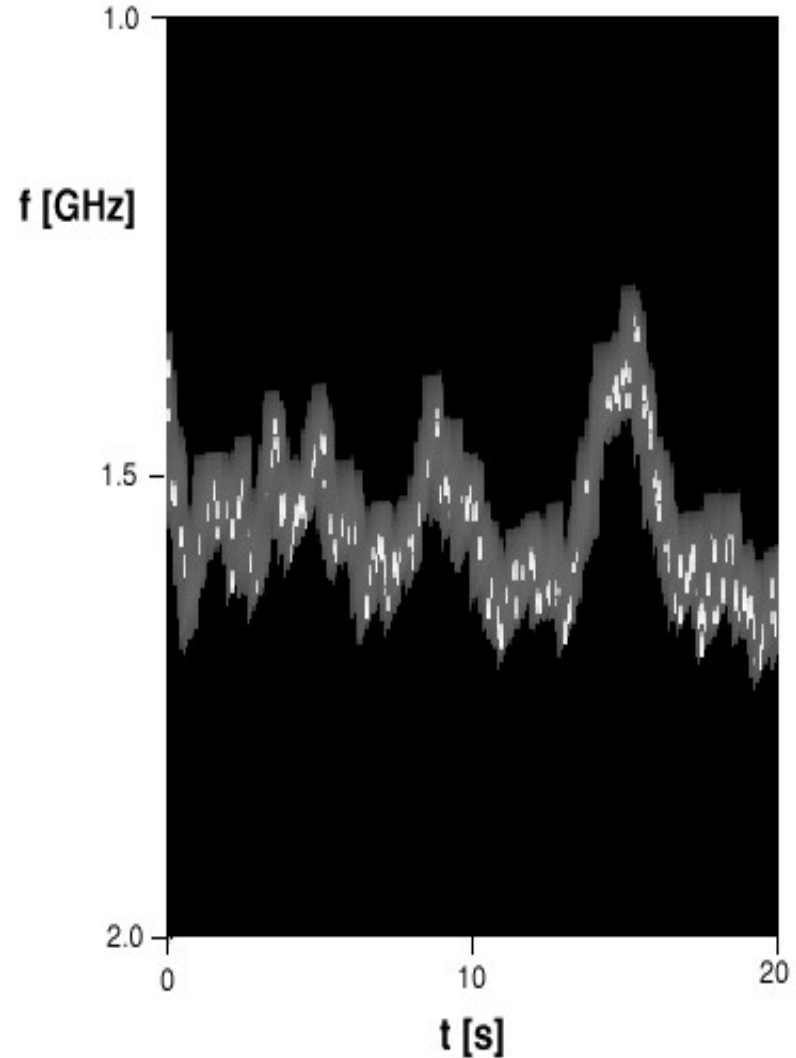
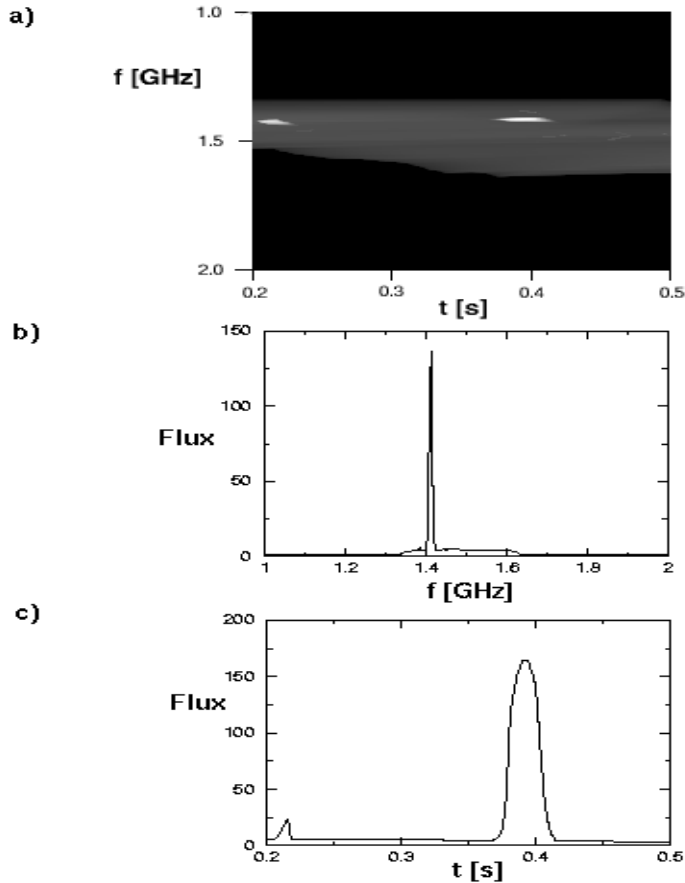
### Application: Plasma turbulence diagnostics

#### What plasma parameters can be estimated:

1. Turbulence level in the jet
2. Energy input scale
3. Energy dissipation rate
4. Power released to kinetic energy

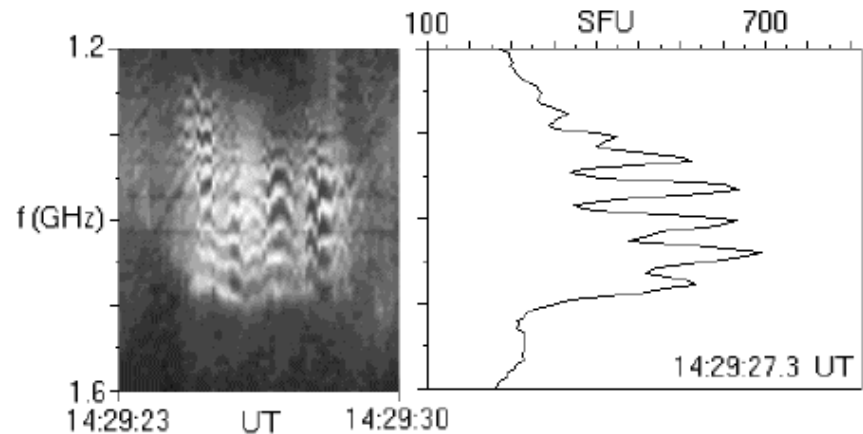
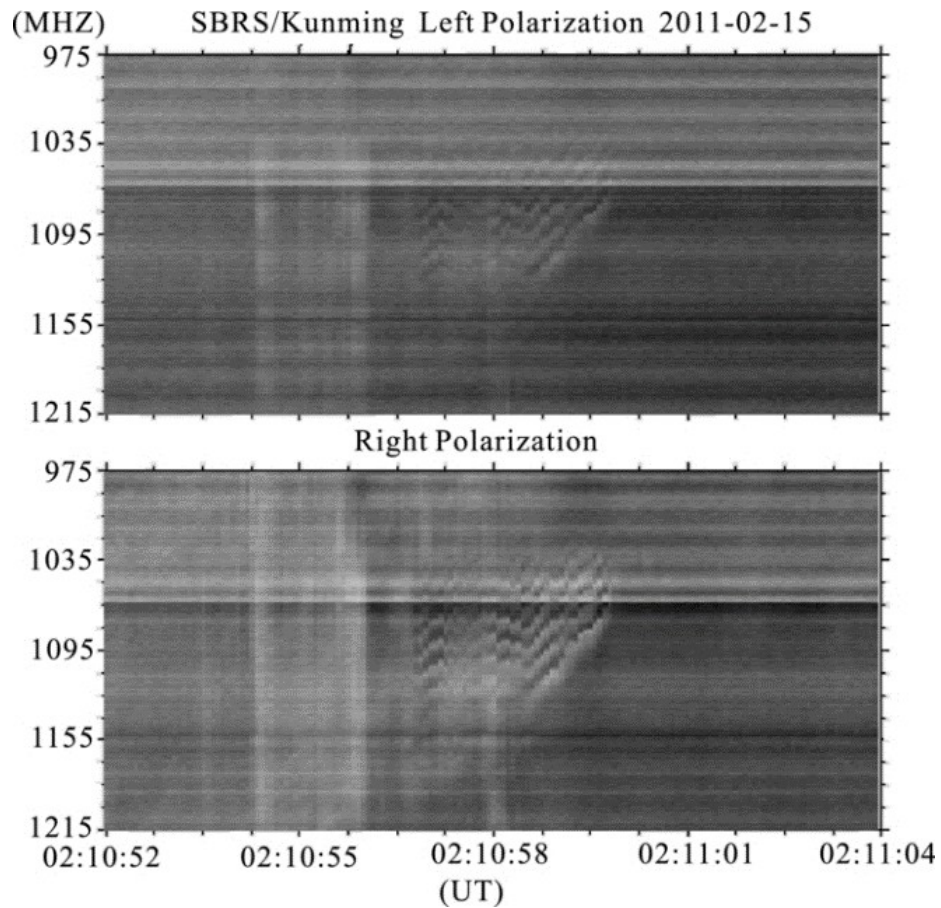
**Radiation from turbulent reconnection jets:**  
Increased turbulence can interrupt emission

dm spikes



Barta & Karlicky 2001, A&A

zebra pattern



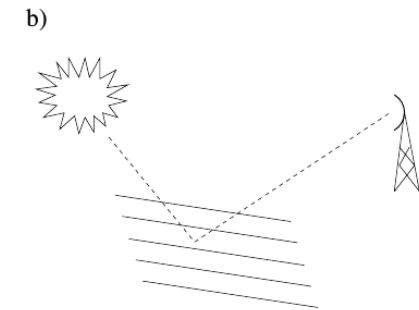
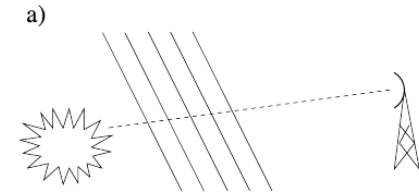
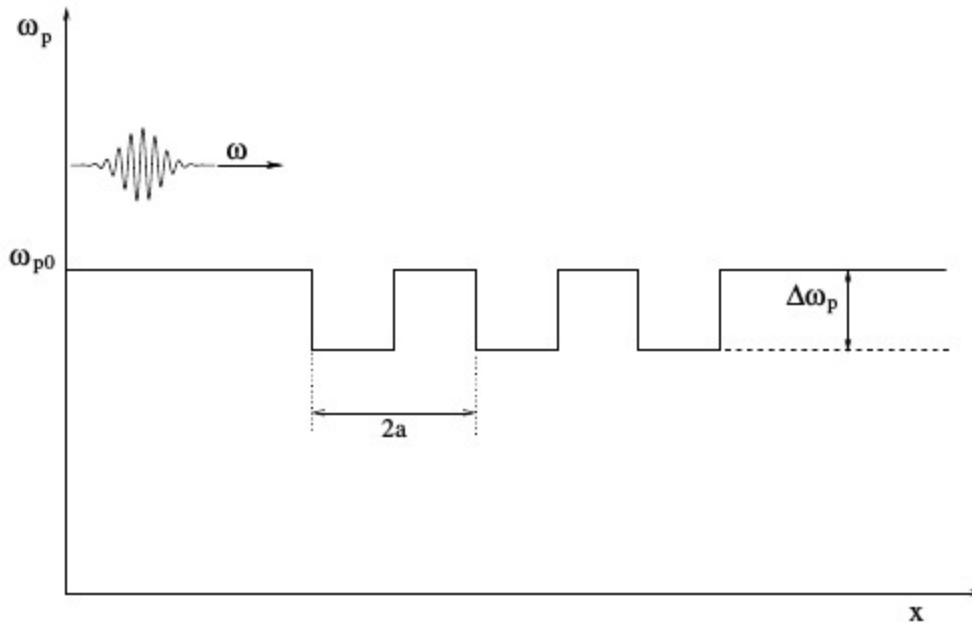
Received radiation pattern is formed also by properties of interleaved plasma!

$$\frac{\partial^2 \mathbf{E}(\mathbf{r}, t)}{\partial t^2} - c^2 \Delta \mathbf{E}(\mathbf{r}, t) + \omega_p^2(\mathbf{r}) \mathbf{E}(\mathbf{r}, t) = 0,$$

where

$$\omega_p^2(\mathbf{r}) = \frac{n(\mathbf{r})e^2}{\epsilon_0 m_e}$$

zebra pattern



Influence of interleaved periodic structures: frequency filtering.  
Similar effect to allowed/forbidden energy belts in crystals.

$$\frac{\partial^2 \mathbf{E}(\mathbf{r}, t)}{\partial t^2} - c^2 \Delta \mathbf{E}(\mathbf{r}, t) + \omega_p^2(\mathbf{r}) \mathbf{E}(\mathbf{r}, t) = 0,$$

where

$$\omega_p^2(\mathbf{r}) = \frac{n(\mathbf{r})e^2}{\epsilon_0 m_e}$$

$$\omega_p(\mathbf{r}) = \omega_{p0} + \delta\omega_p(\mathbf{r}).$$

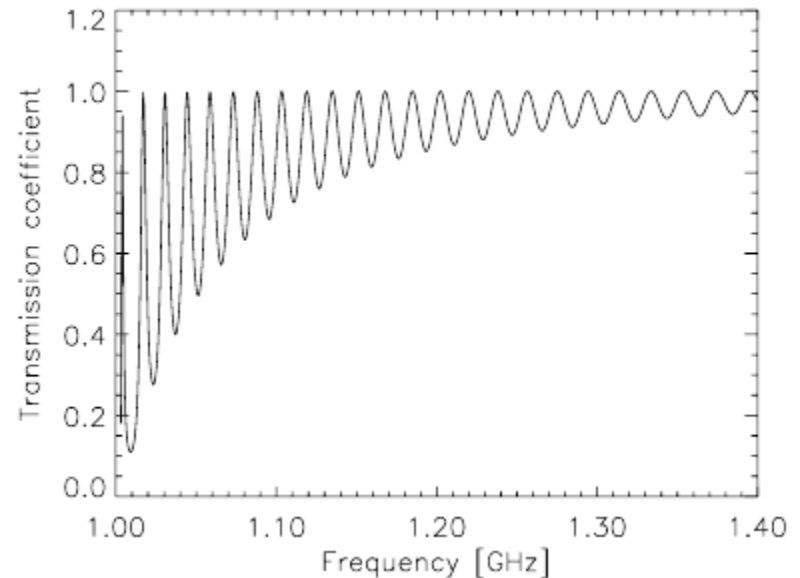
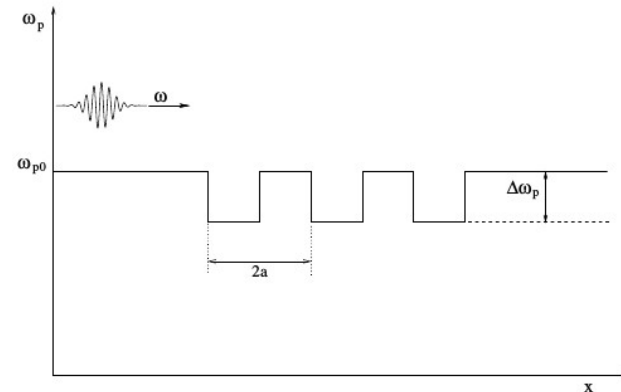
$$m = \frac{\hbar\omega_p}{c^2}.$$

$$i\hbar \frac{\partial \mathbf{E}}{\partial t} = -\frac{\hbar^2 \Delta}{2m_0} \mathbf{E} + (V(\mathbf{r}) + m_0 c^2) \mathbf{E}$$

with

$$V(\mathbf{r}) = \hbar \delta\omega_p(\mathbf{r}).$$

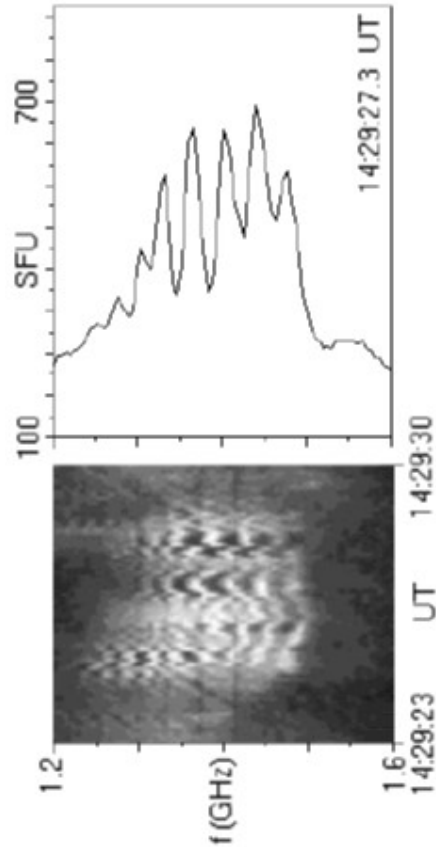
zebra pattern



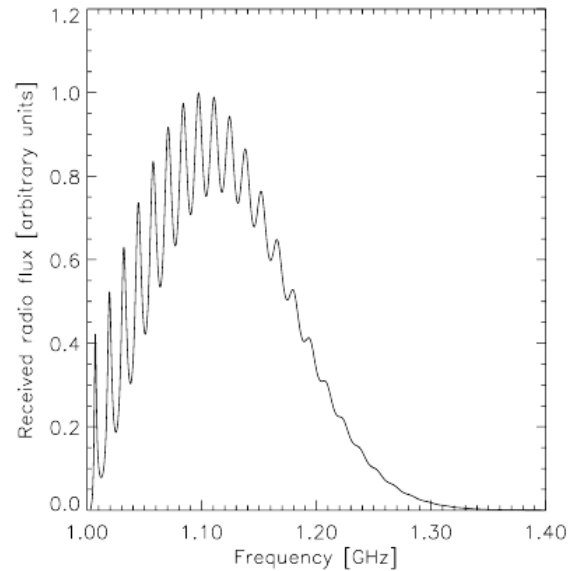
Influence of interleaved periodic structures: frequency filtering.  
Similar effect to allowed/forbidden energy belts in crystals.

zebra pattern

Observation



Model



**Barta & Karlicky 2006, A&A**



## Solar radio dynamic spectroscopy

Calibration of spectral data

$$P_{rec}(f) = G(f) \cdot B(f) + P_{sys}(f)$$

## Methods of calibration

- ▶ Absolute: Hot and ambient loads – noise generators (heated resistors noise diode, ...)
- ▶ Relative: Scaled to an (celestial) object with known radio brightness – *calibrator*  
Typical radio flux calibrators: Quasars, solar system objects (planets & their moons)

For spectroscopic data all quantities are frequency dependent.

Finding  $G=G(f)$  and  $T_{sys}=T_{sys}(f)$  from observed calibrators with known spectrum

→ **bandpass calibration**

For dynamic radio spectra of solar bursts the *Quiet Sun Radiation* (QSR) with well known spectral properties is used as calibrator:

$$G(f) = \frac{P_{obs}^{QSR}(f) - N(f)}{P_{teor}^{QSR}(f)} \quad \longrightarrow \quad F_{cal}(f) = \frac{QSR_{teor}(f)}{QSR_{meas}(f) - N(f)} \cdot [F_{meas}(f) - N(f)]$$

## Solar radio dynamic spectroscopy

### Calibration of spectral data

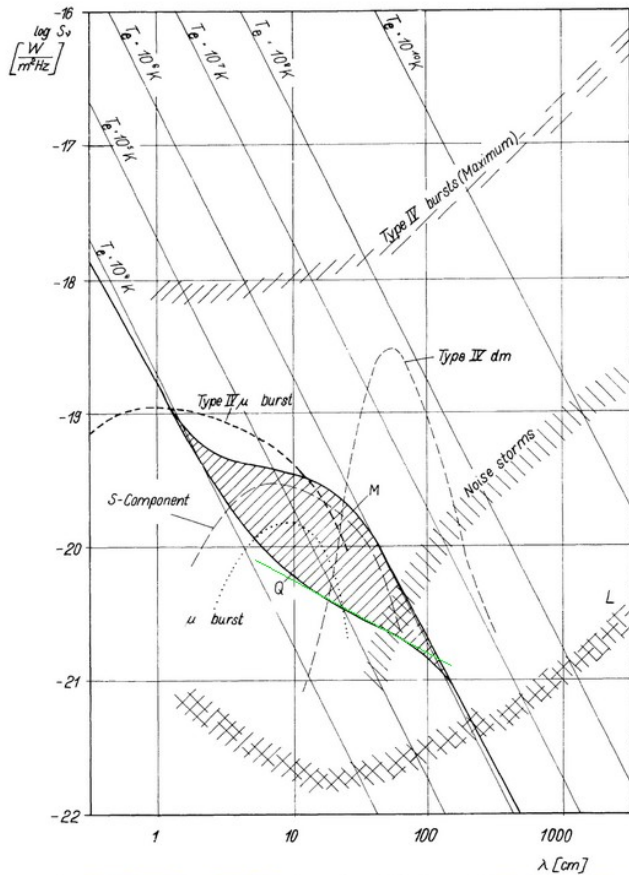
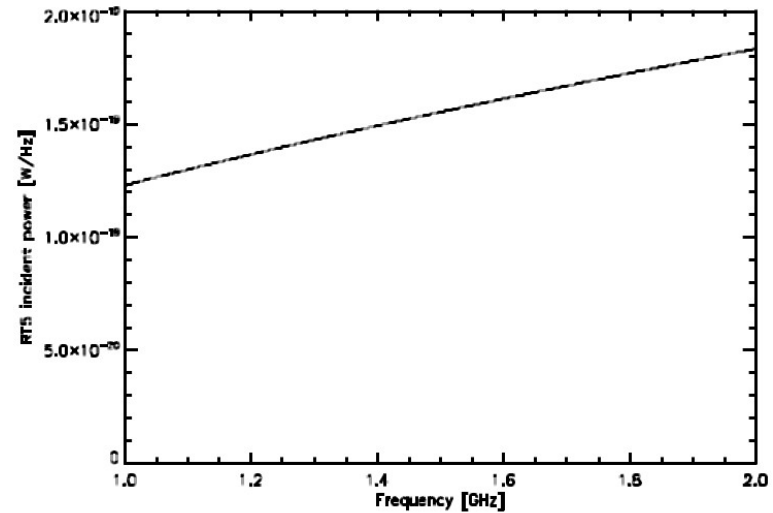


Fig. III.1. Solar radio spectra: Q – ‘quiet Sun’, M – maximum daily flux; L – limit of detectability of single-frequency patrol observations (after Krüger, 1968).



$$\frac{P_{QSR\ Inc}(f)}{W\ Hz^{-1}} = 1,23 \times 10^{-19} \cdot \left(\frac{f}{GHz}\right)^{0,577}$$

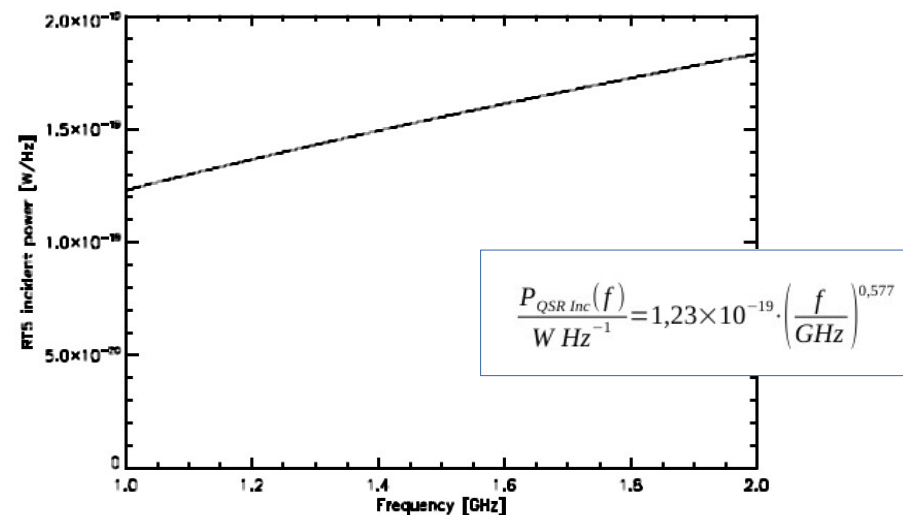
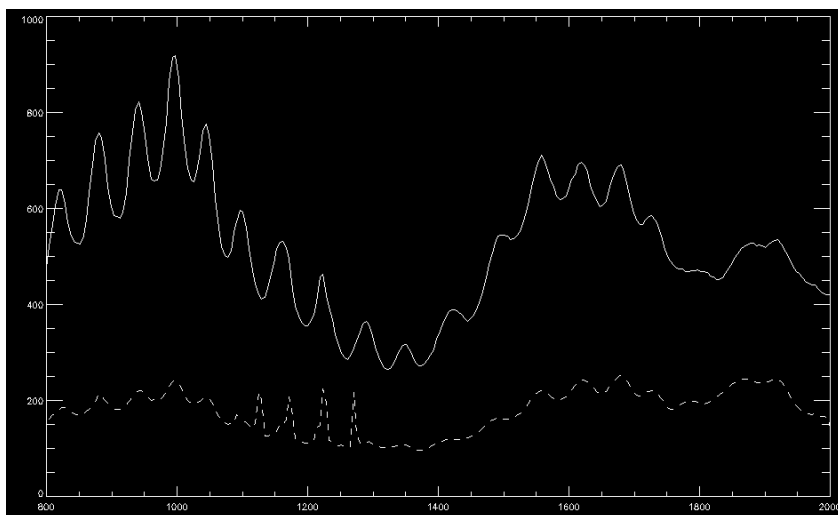
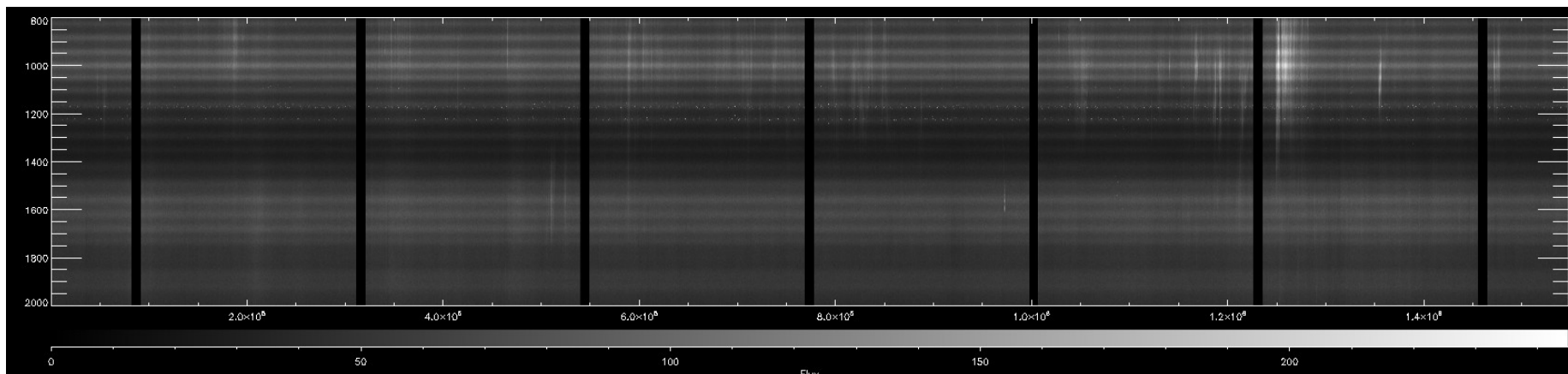
$$F_{cal}(f) = \frac{QSR_{teor}(f)}{QSR_{meas}(f) - N(f)} \cdot [F_{meas}(f) - N(f)]$$

# Methods of radio astronomy

## Solar radio dynamic spectroscopy

## Calibration of spectral data

Uncalibrated data

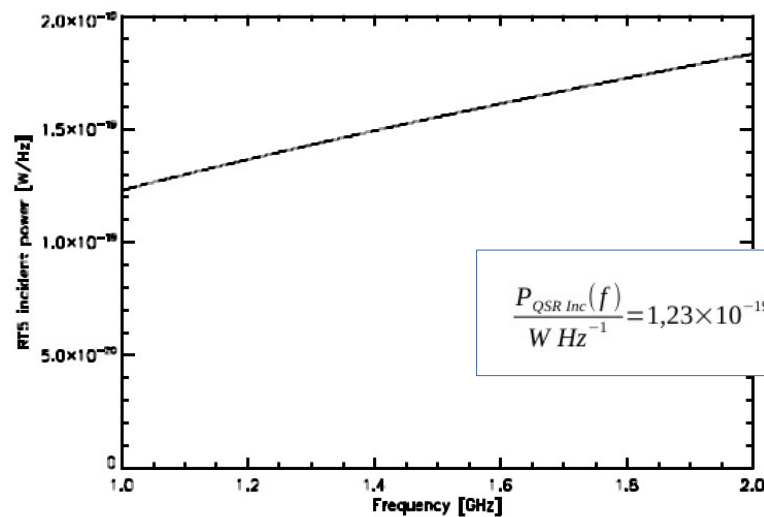
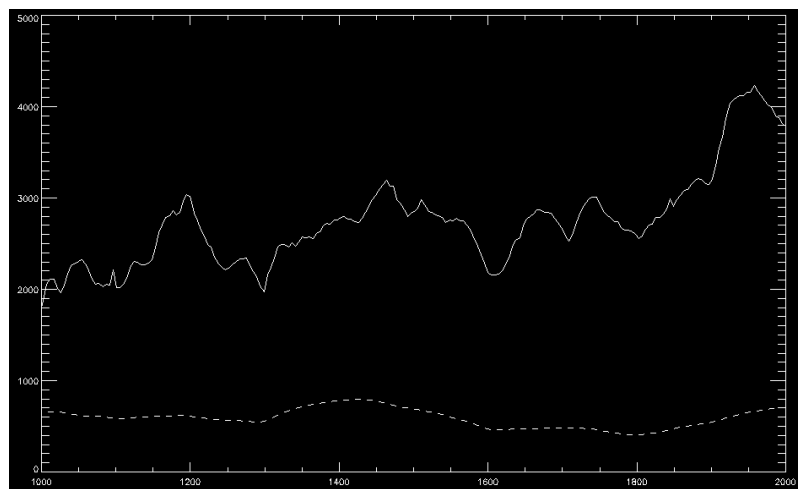
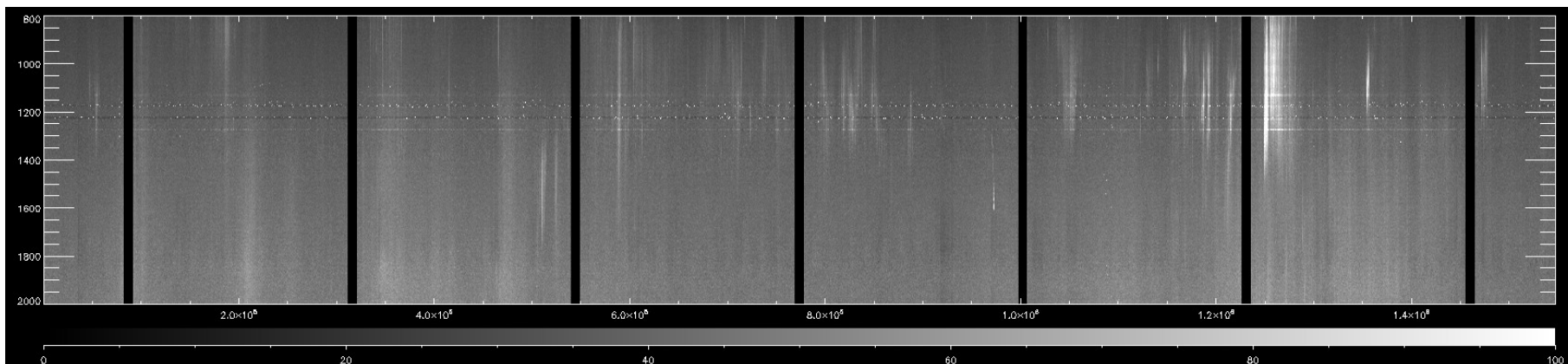


# Methods of radio astronomy

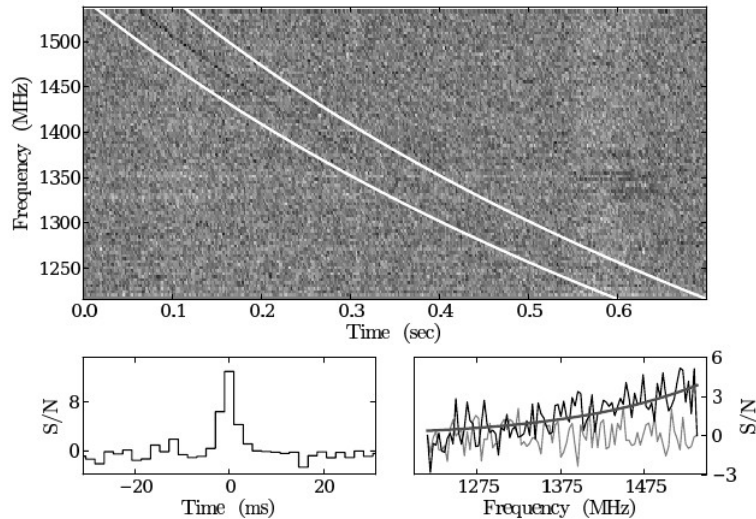
Solar radio dynamic spectroscopy

Calibration of spectral data

Calibrated data

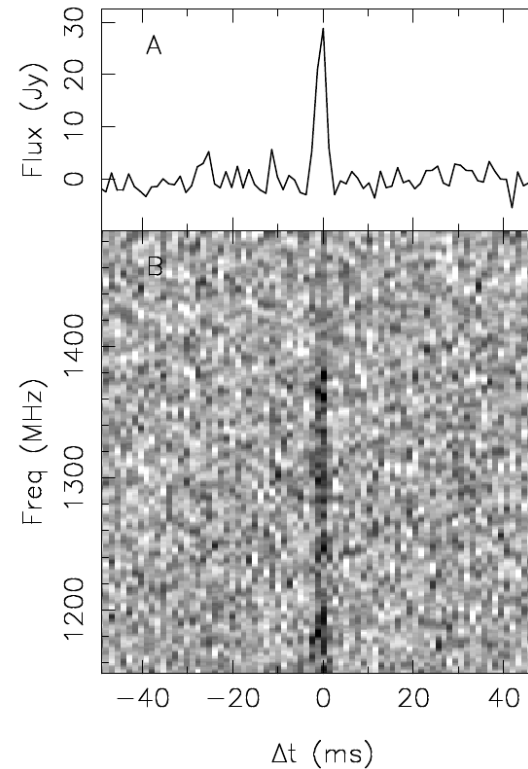


## Radio dynamic spectroscopy / spectropolarimetry: Non-solar applications



**Figure 2.** Characteristic plots of FRB 121102. In each panel the data were smoothed in time and frequency by a factor of 30 and 10, respectively. The top panel is a dynamic spectrum of the discovery observation showing the 0.7 s during which FRB 121102 swept across the frequency band. The signal is seen to become significantly dimmer towards the lower part of the band, and some artifacts due to RFI are also visible. The two white curves show the expected sweep for a  $\nu^{-2}$  dispersed signal at a  $DM = 557.4 \text{ pc cm}^{-3}$ . The lower left panel shows the dedispersed pulse profile averaged across the bandpass. The lower right panel compares the on-pulse spectrum (black) with an off-pulse spectrum (light gray), and for reference a curve showing the fitted spectral index ( $\alpha = 10$ ) is also overplotted (medium gray). The on-pulse spectrum was calculated by extracting the frequency channels in the dedispersed data corresponding to the peak in the pulse profile. The off-pulse spectrum is the extracted frequency channels for a time bin manually chosen to be far from the pulse.

## Fast Radio Bursts / FRBs



FRB.gif

### Fast Radio Bursts / FRBs

$$\omega^2 = \omega_{pe}^2 + k^2 c^2$$

$$v_g = \frac{\partial \omega}{\partial k}$$

$$v_g = c \sqrt{1 - \frac{\omega_p^2}{\omega^2}}$$

$$\frac{1}{v_g} = \frac{1}{c} \left( 1 + \frac{1}{2} \frac{v_p^2}{v^2} \right) \quad \longrightarrow \quad \tau_D = \int_0^L \frac{dl}{v_g} \cong \frac{1}{c} \int_0^L \left( 1 + \frac{1}{2} \left( \frac{v_p}{v} \right)^2 \right) dl = \frac{1}{c} \int_0^L \left( 1 + \frac{e^2}{2\pi m_e} \frac{1}{v^2} N(l) \right) dl$$

$$\tau_D = \frac{L}{c} + \frac{e^2}{2\pi c m_e} \frac{1}{v^2} \int_0^L N(l) dl.$$

$$\Delta \tau_D = \frac{e^2}{2\pi c m_e} \left[ \frac{1}{v_1^2} - \frac{1}{v_2^2} \right] \int_0^L N(l) dl$$



$$\Delta \tau_D = \frac{e^2}{2\pi c m_e} \left[ \frac{1}{v_1^2} - \frac{1}{v_2^2} \right] \int_0^L N(l) dl$$

$$DM = \int_0^\infty \left( \frac{N}{\text{cm}^{-3}} \right) d \left( \frac{l}{\text{pc}} \right)$$

Dispersion measure

### Fast Radio Bursts / FRBs

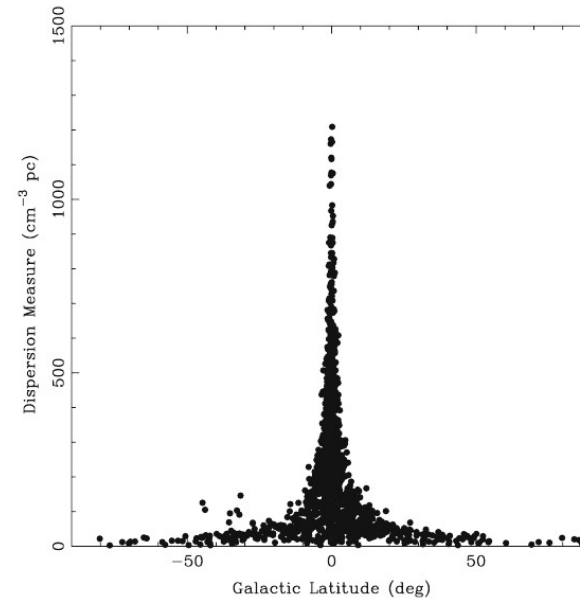
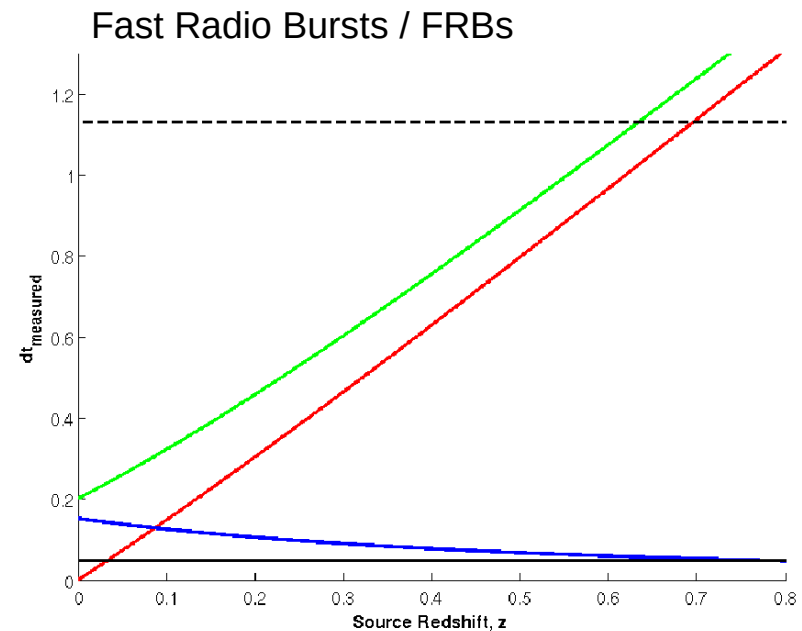


Fig. 2.3 Dispersion measure, DM, for pulsars at different galactic latitudes [adapted from B. Klein (MPIfR) unpublished]

## Radio dynamic spectroscopy / spectropolarimetry: Non-solar applications

$$\Delta \tau_D = \frac{e^2}{2\pi c m_e} \left[ \frac{1}{v_1^2} - \frac{1}{v_2^2} \right] \int_0^L N(l) dl$$

$$DM = \int_0^\infty \left( \frac{N}{\text{cm}^{-3}} \right) d \left( \frac{l}{\text{pc}} \right)$$

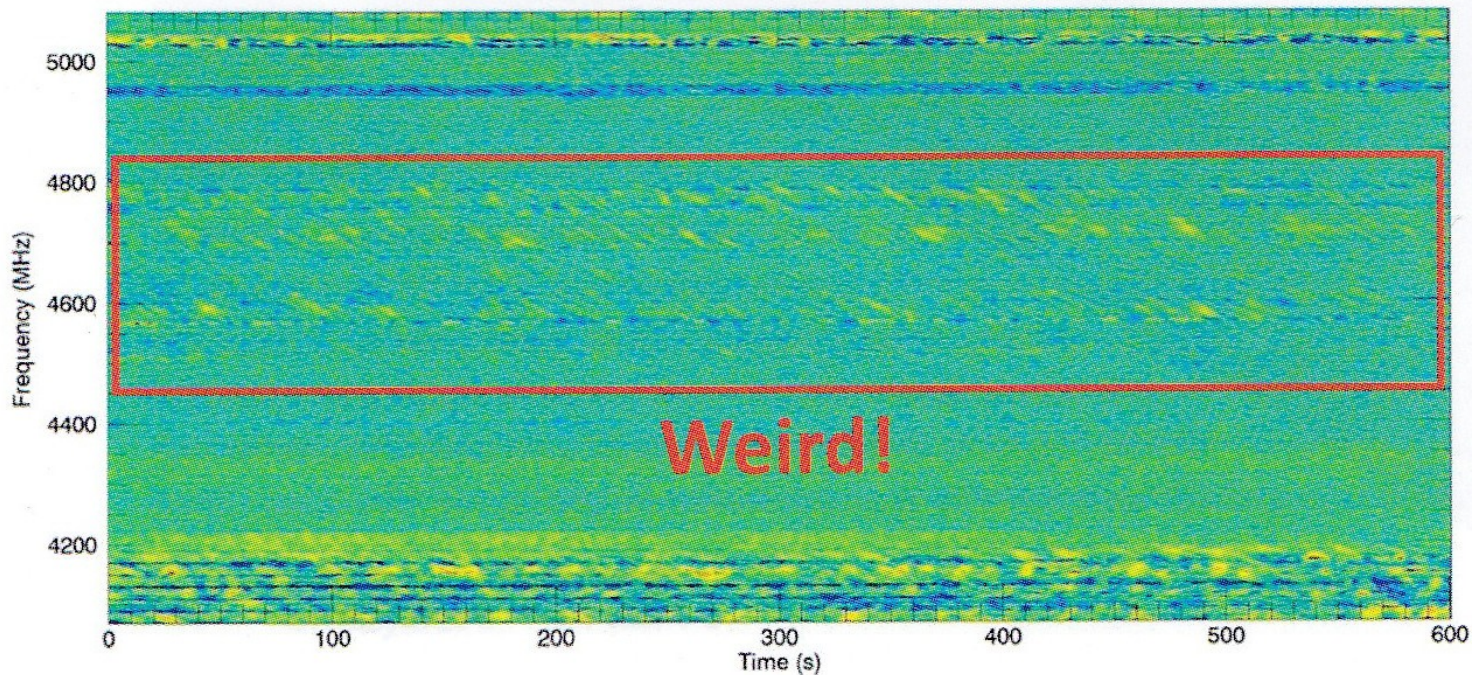


**Figure 5.4** – Modelled dispersive delay (in seconds) across the observing band of the HTRU survey is shown plotted against redshift. The flat dashed line indicates the delay across the observing band for FRB 110220. The solid black line is the MW contribution and is constant irrespective of the redshift of the source, the red line is the delay due to the IGM, the blue line is the delay contribution from a host galaxy with  $DM_{\text{Host}} = 100 \text{ cm}^{-3} \text{ pc}$ , and the green line is the sum of the IGM, host and MW contributions. When the sum of contributions is equal to the measured delay the redshift is inferred.

**Weird signal @ Arecibo (Astropis 2017/2)**

# Ross 128

57886.037454 MJD - 2017/05/13 00:53:55 UTC - 2017/05/12 20:53:55 AST



$$k_{\pm}^2 = \frac{\omega^2}{c^2} \left( 1 - \frac{\omega_p^2}{\omega(\omega \pm \omega_c)} \right) \quad \text{Ordinary/extraordinary ELMG wave in magnetised plasma}$$

$$2\Delta\psi = (k_+ - k_-)\Delta z$$

$$\Delta\psi = \frac{\omega_p^2 \omega_c}{2c \omega^2} \Delta z = \frac{2\pi N e^3 B}{m^2 c \omega^2} \Delta z$$

$$\Delta\psi = \frac{e^3}{2\pi m^2 c} \frac{1}{\nu^2} \int_0^L B_{\parallel}(z) N(z) dz$$

$$\begin{aligned} \frac{\text{RM}}{\text{rad m}^{-2}} &= 8.1 \times 10^5 \int_0^{L/\text{pc}} \left( \frac{B_{\parallel}}{\text{Gauss}} \right) \left( \frac{N_e}{\text{cm}^{-3}} \right) d \left( \frac{z}{\text{pc}} \right) \\ &= \frac{\left( \frac{\Delta\psi_1}{\text{rad}} \right) - \left( \frac{\Delta\psi_2}{\text{rad}} \right)}{\left( \frac{\lambda_1}{\text{m}} \right)^2 - \left( \frac{\lambda_2}{\text{m}} \right)^2} \end{aligned}$$

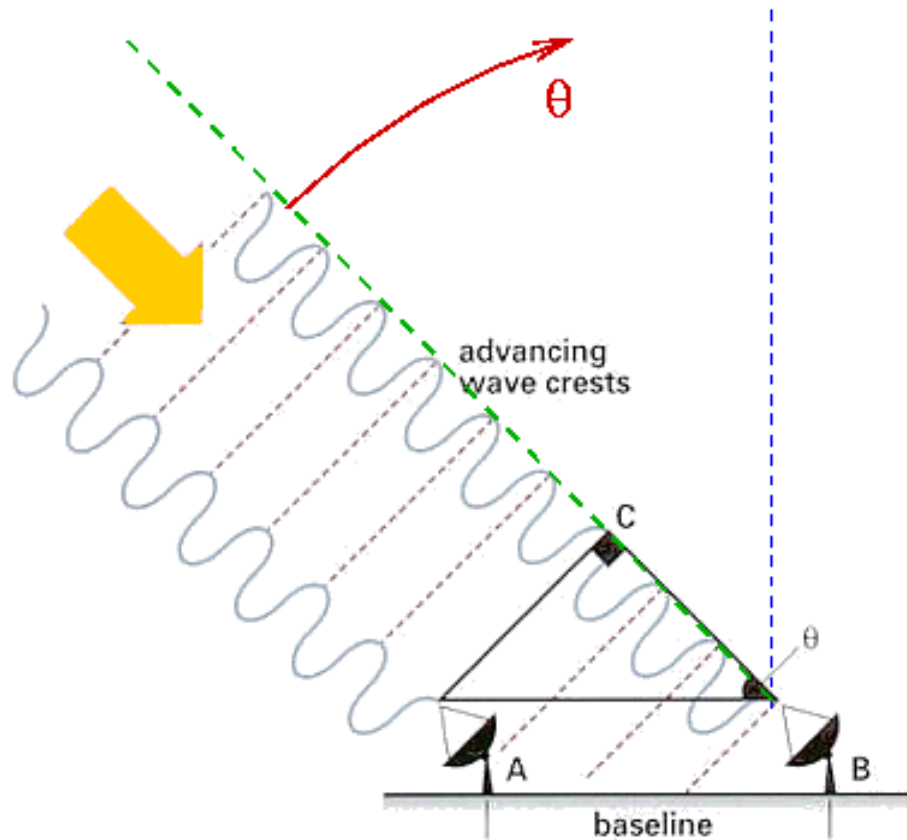
Overview on interferometry  
(main lecture next time)

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# Methods of radio astronomy

“Old-fashion” radio interferometry: Phased arrays - **analog signal sum**

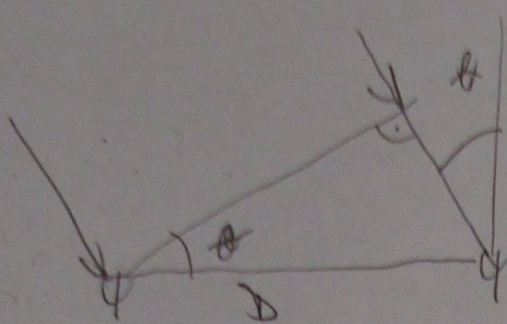


Known result from  
a **double-slit** experiment:

$$|I_{AB}(\theta)| = 1 + \cos \frac{2\pi D}{\lambda} \theta$$

# Methods of radio astronomy

“Old-fashion” radio interferometry: Phased arrays - **analog signal sum**



$$E_A = E_0 \cdot e^{i\omega t}$$

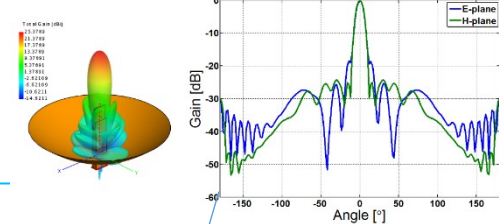
$$E_B = E_0 \cdot e^{i\omega(t - \frac{D \sin \theta}{c})}$$

$$I = |E_A + E_B|^2 = E_0^2 \cdot \left( e^{i\omega t} + e^{i\omega(t - \frac{D \sin \theta}{c})} \right) \cdot \left( e^{-i\omega t} + e^{-i\omega(t - \frac{D \sin \theta}{c})} \right)$$
$$= E_0^2 \left( 2 + e^{i\omega \tau} + e^{-i\omega \tau} \right) = 2E_0^2 \cdot (1 + \cos \omega \tau)$$

$$\tau = \frac{D \sin \theta}{c} \quad \theta \ll 1 \quad \sin \theta \approx \theta$$

$$\omega \tau = 2\pi f \frac{D \cdot \theta}{c} = \frac{2\pi D}{\lambda} \theta$$

$$I = 2E_0^2 \left( 1 + \cos \frac{2\pi D}{\lambda} \theta \right)$$

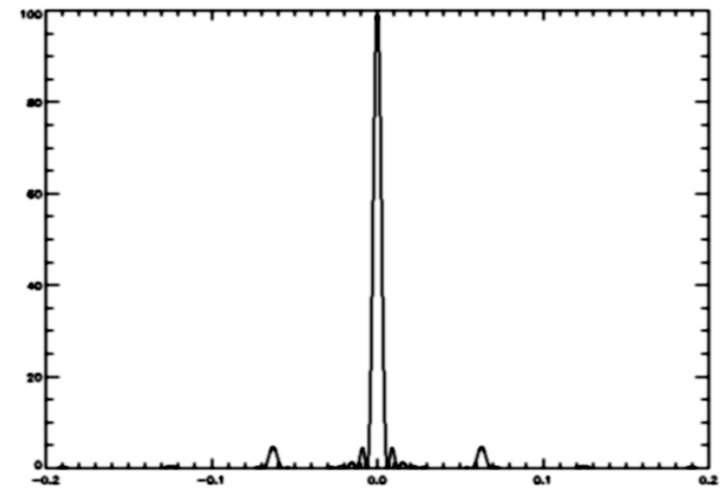
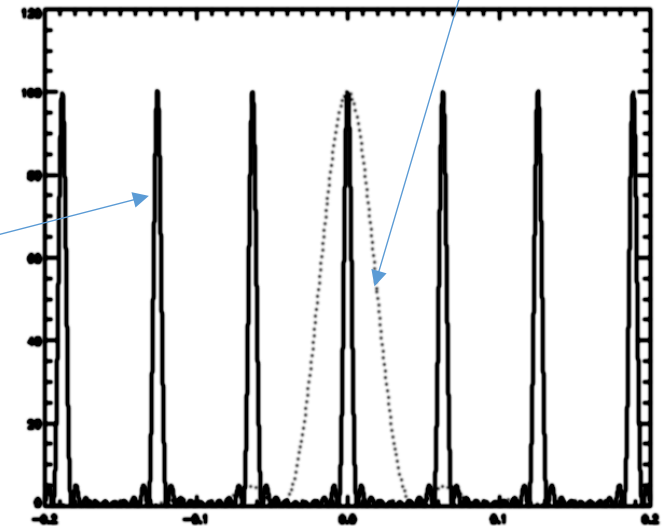
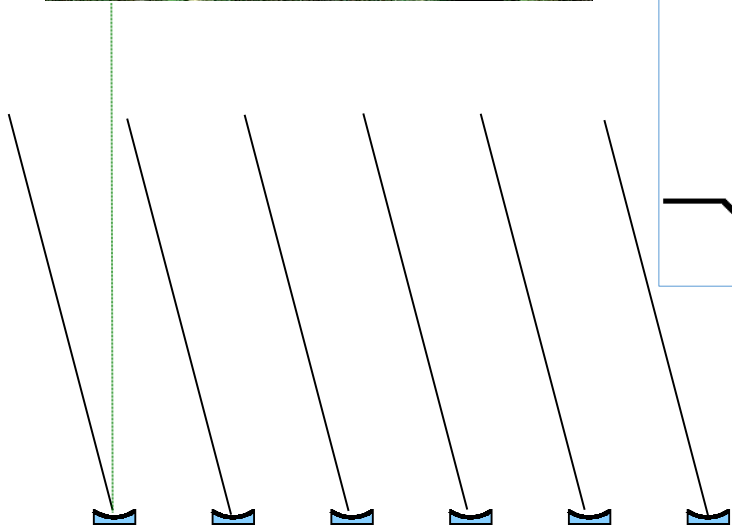
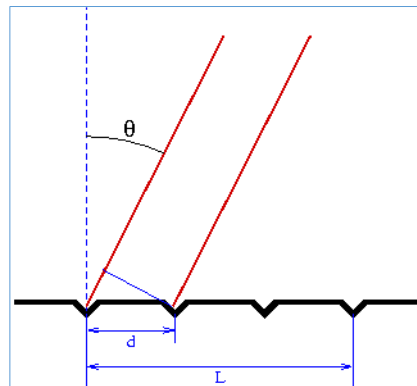


### Old good times – sparse antenna rows / phased arrays (beam forming)



Analogy with **optical gratings** used in spectrographs

$$|I_{\Sigma}(\theta)| = \frac{1 - \cos N \frac{2\pi D}{\lambda} \theta}{1 - \cos \frac{2\pi D}{\lambda} \theta}$$





# Methods of radio astronomy

“Old-fashion” radio interferometry: Phased arrays - **analog signal sum**

Diagram illustrating a linear array of antennas with spacing  $D$ . The path difference between antennas is labeled as  $r = 0, r = D, r = 2D, \dots, r = (N-1)D$ .

The intensity  $I$  is given by the magnitude squared of the sum of the electric field contributions from each antenna:

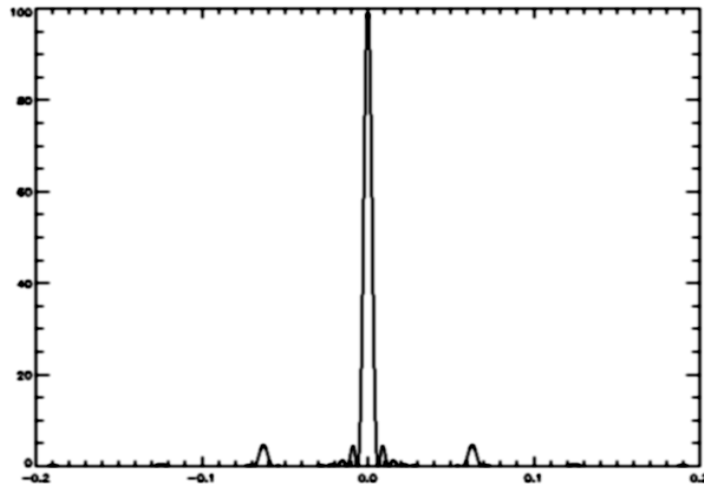
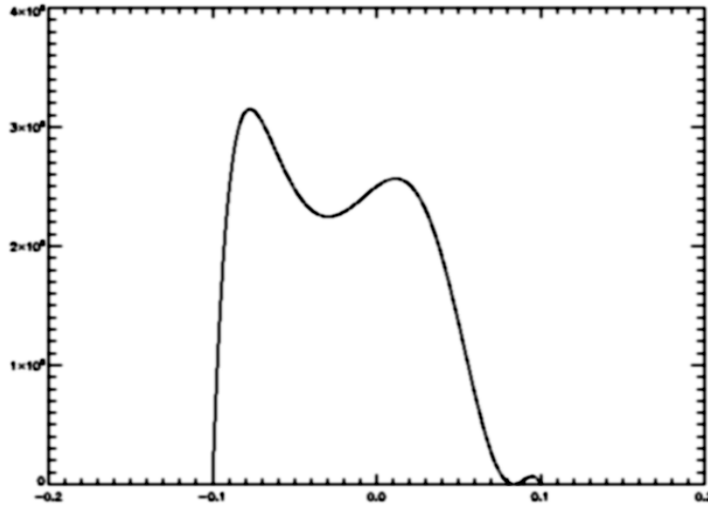
$$I = \left| \sum_{r=0}^{N-1} E_0 e^{i\omega(t - r)} \right|^2 = \left| E_0 \cdot e^{i\omega t} \cdot \sum_{r=0}^{N-1} \left( e^{-i\omega r} \right)^r \right|^2$$

$$= E_0^2 \cdot \left| \frac{(e^{-i\omega r})^N - 1}{e^{-i\omega r} - 1} \right|^2 = N E_0^2 \frac{1 - \cos N\theta}{1 - \cos \theta}$$

The final result is boxed in green:

$$I = N E_0^2 \frac{1 - \cos N \frac{2\pi D}{\lambda} \theta}{1 - \cos \frac{2\pi D}{\lambda} \theta}$$

## Old good times – sparse antenna rows/ phased arrays



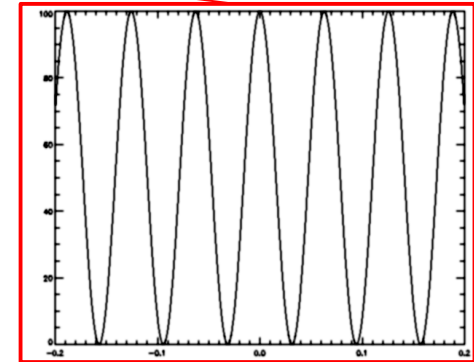
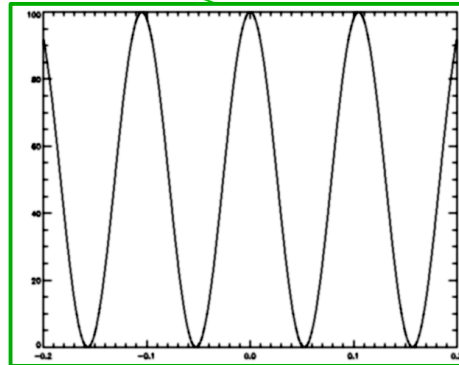
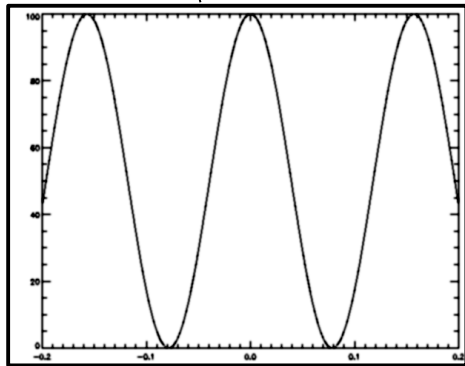
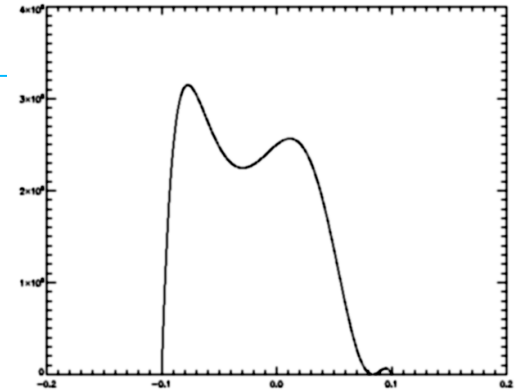
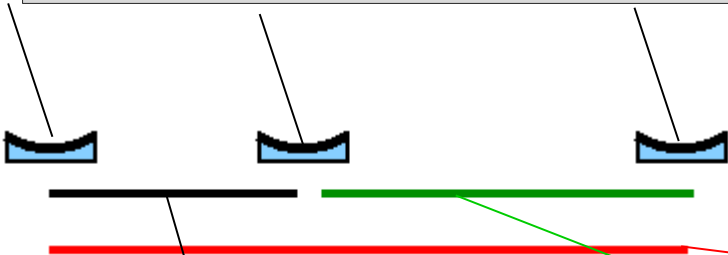
- Scanning with the beam
- Frequently supplied by MFI





# Methods of radio astronomy

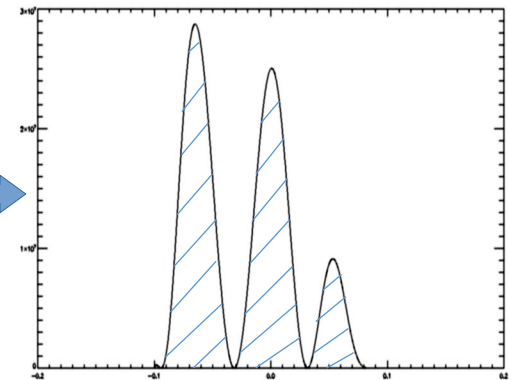
Modern systems - **digital signal product**

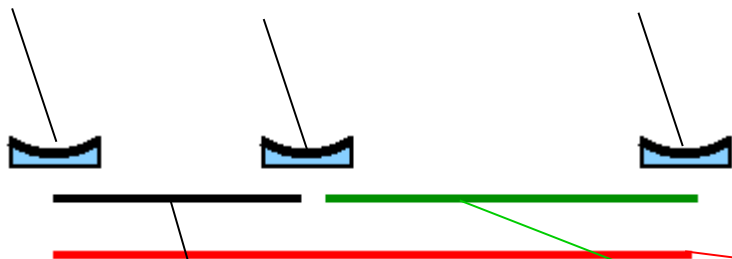


**Nowadays: Aperture synthesis – decomposition of image to harmonics = Fourier transform**

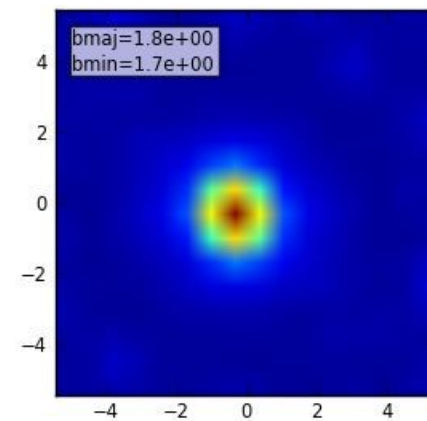
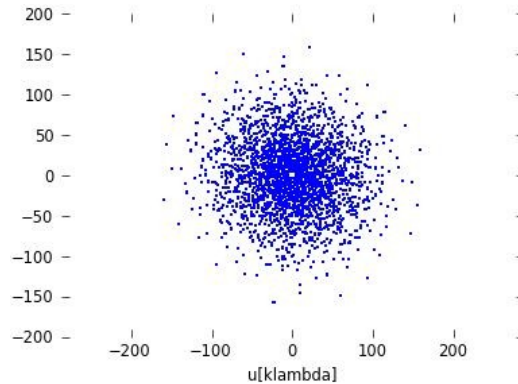
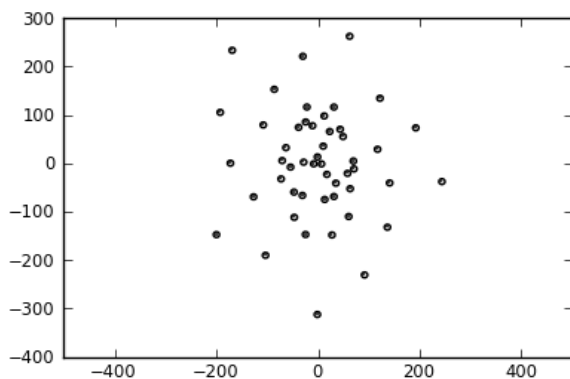
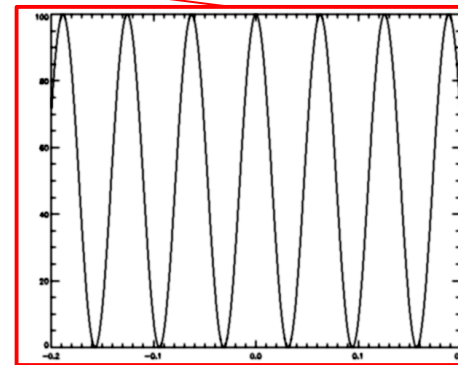
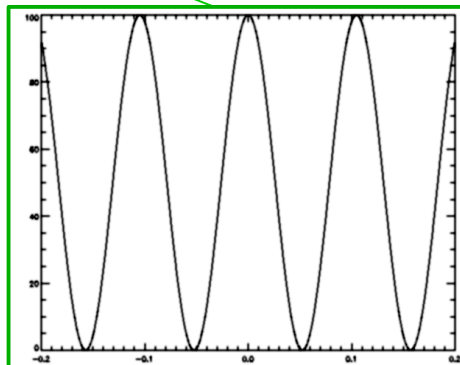
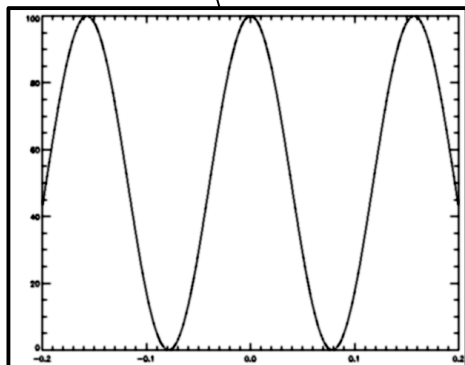
$$\langle P_{12}(D/\lambda) \rangle = \langle E_1 \cdot \overline{E_2} \rangle = G \cdot \int_{-\infty}^{+\infty} B(\theta) \exp\left(i \frac{2\pi D}{\lambda} \theta\right) d\theta$$

Cross-correlations → **interferometric visibilities**





$$\langle I_{AB} \rangle(\theta) = \int_{-\infty}^{+\infty} B(\theta) \exp\left(i \frac{2\pi D}{\lambda} \theta\right) d\theta$$



### Response to the point source

2 antennas

3 antennas

8 antennas x 240 samples

**Much more comes in the next lecture!**

